Active and Action Based SLAM

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Introduction

- So Far:
  - Introduced to the fundamental problem of SLAM
  - Probabilistic Formulation and the EKF
  - Building Large Maps

- SLAM: use in mobile robotics applications

- What about the motion of the robotic platform and the features observed? What effect does this have on SLAM?
Simultaneous Localisation And Mapping (SLAM)

- Start at unknown location with no a priori map information.
- Predict motion through DR
- Make relative observations to local features and build a map through these observations.
- Predict and re-observe features which are in the map and begin to correlate
- Correlation assists in constraining drift in inertial solution
- Update the vehicle and feature estimates at each observation
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Correlations Between Features
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All Correlated to Vehicle also
“Closing the Loop” in SLAM

- **Vehicle now returns to re-observe features not seen for a while**
- **Network of correlations serves to reduce the uncertainty of both feature and vehicle location estimates**
- **How much this uncertainty is reduced is strongly coupled to vehicle actions taken, order of features observations and the size of the loop**
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SLAM from a Control-Theoretic Perspective

- **SLAM Context:**
  External navigation aids (GPS) or prior terrain map unavailable or unreliable

- **Vehicle control actions now have a significant effect on Localisation and Mapping**
Active SLAM

- Coupling control actions into the estimation process: **Active SLAM**
- Understand the connection between localisation, mapping and control
- Design intelligent control strategies for maximising localisation and mapping performance
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- Information-Based Active SLAM
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- One commonly used approach:
  - **Information-Theoretic Control**
    - Information measures quantify the certainty or uncertainty in a probabilistically represented estimate
    - Information measures used as a cost function for potential control actions
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Information-Based Cost/Utility Functions for Active SLAM

- Probability distribution represents SLAM state estimate
Information-Based Cost/Utility Functions for Active SLAM

Example:

State Vector  \( \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \)

Estimate Represented by Probability Distribution  \( p(x) \)
Information-Based Cost/Utility Functions for Active SLAM

- Probability distribution represents SLAM state estimate
- Our information metric represents the uncertainty in our probability distribution

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Estimate Represented by Probability Distribution $p(\mathbf{x})$

$$ I[\mathbf{x}] = f[p(\mathbf{x})] $$

- The information metric is a function of the probability distribution
Example: let \( p(x) \) be Gaussian

\[
p(x) \sim (\hat{x}, P) \text{ mean covariance}
\]
Example: let $p(x)$ be Gaussian

$$p(x) \sim \left( \hat{x}, \begin{pmatrix} \mathbf{P} \end{pmatrix} \right)$$

Some Information metrics:

- $H(x)$ (Entropy): Related to the volume of the covariance ellipse
- $\text{Trace}(P)$: Related to the size of the covariance ellipse

$$h(x) = \frac{1}{2} \log((2\pi e)^n |\mathbf{P}|)$$
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1-Sigma Confidence Ellipse

- Entropy $H(x) \propto \text{Volume}$
- $\text{Trace}(P) = \sum \text{eigs}(P)$
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In active SLAM we are interested in information gain associated with a particular action.

Information gain is defined as the difference in the information of our estimate before and after a particular action “a” is taken:

\[
\mathcal{I}[x, a] = f[p(x|a)] - f[p(x)]
\]
In order to evaluate the information gain associated with a proposed action "a", we must approximate the expected estimate probability distribution $p(x|a)$ at the end of the action.
Estimating P(x|a)

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- **EKF Example**: approximating p(x|a) is equivalent to approximating the covariance matrix P at the end of the action.
Estimating $P(x|a)$

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- **EKF Example**: approximating $p(x|a)$ is equivalent to approximating the covariance matrix $P$ at the end of the action.

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**EKF Prediction**

$$P_{k+1}^- = \nabla F P_k \nabla F^T + \nabla G Q_k \nabla G^T$$

**EKF Update**

$$S_{k+1} = \nabla H P_{k+1}^- \nabla H^T + R_{k+1}$$

$$W_{k+1} = P_{k+1}^- \nabla H^T S_{k+1}^{-1}$$

$$P_{k+1}^+ = P_{k+1}^- - W_{k+1} S_{k+1} W_{k+1}^T$$
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**Recursive Ricatti Equation**

$$P(k+1) = \nabla F_{k+1} [P(k) - P(k) \nabla H_k^T (\nabla H_k P(k) \nabla H_k^T + R)^{-1} \nabla H_k P(k)] \nabla F_{k+1}^T$$

$$+ \nabla G_{k+1} Q \nabla G_{k+1}^T$$

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• Information-Based Active SLAM

• Extending Active SLAM to multiple objectives

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Single Vehicle Active SLAM Problem

- Choose a trajectory that maximises $\mathcal{I}[\mathbf{x}, \mathbf{a}]$
- Propose several trajectories
- Estimate the observations that will be made along each trajectory
- Estimate the value of the covariance matrix at the end of each trajectory
- Compute $\mathcal{I}[\mathbf{x}, \mathbf{a}]$ for each trajectory and fly optimal path
Single Vehicle Active SLAM Problem

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![Diagram of single vehicle active SLAM problem](image-url)
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Covariance Before Trajectory and Observations
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Information-Theoretic Active SLAM Control Architecture

Vehicle

Collect Potential Actions \( A = \{ a_1, a_2, \ldots, a_n \} \)

SLAM Localisation and Map Estimates

Calculate Post-action Estimate Distribution \( p(x|a) \) for Each

Maximise Utility Function

Calculate Information Utility \( I[x,a] \) for Each

Trajectory Command \( a^* \) sent to Control System, Process Repeated when Trajectory is Complete
Some Issues

- Two main issues with an information-theoretic approach:
  
  - **Computation of information utility:** the computation of the information gain relies on an approximation of $p(x|a)$, complexity grows in the same way as SLAM!
  
  - **Size of action space:** For more complex problems in robotics the number of potential actions to consider may be very large
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Active SLAM and other Objectives

- In real applications other objectives will usually drive the SLAM mission
- Mission objectives may include:
  - Building a high accuracy map of an unknown area
  - Navigate and localise a robot in an unknown/partially unknown area
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Mission objectives may include:
- Building a high accuracy map of an unknown area
- Navigate and localise a robot in an unknown/partially unknown area

Other objectives/constraints that may be considered additionally to information gain:
- Minimise exploration/travel time
- Maximise area coverage
- Constrain localisation error build up
Active SLAM Example Scenario

Objective Destination
Active SLAM Example Scenario

Prior Map Information

Objective Destination

Prior Map Information
Objective: Navigate across the terrain to the objective in a time-efficient manner, while maintaining localisation accuracy above a given threshold
Active SLAM Example Scenario

Objective Destination
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Prior Map Information

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Prior Map Information

New Features

Prior Map Information

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Active and Action Based SLAM
Active SLAM Example Scenario

Prior Map Information

Localisation Uncertainty

Objective Destination

Prior Map Information
Active SLAM Example Scenario

Prior Map Information

Localisation Uncertainty Grows

Objective Destination

Prior Map Information
Active SLAM Example Scenario

Prior Map Information

Move to Make Observations of Prior Map Features

Objective Destination

Prior Map Information
Active SLAM Example Scenario

Prior Map Information

Close the loop on Mapped Features

Objective Destination

Prior Map Information
Active SLAM Example Scenario

- How do we balance between exploring new terrain and moving to prior features?

Prior Map Information

Objective Destination

Prior Map Information
Active SLAM Example Scenario

- Develop a path planning system where this balance is made.
Active SLAM Example Scenario

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Approach

- Produce Paths to destination

- Predict Localisation performance along path
  - Using a Ricatti Equation approximate the localisation and mapping information along the path
  - Problem: How do we predict localisation performance over unknown terrain?

- Find shortest path where localisation errors are constrained within given bounds
SLAM Performance over Unknown Terrain

- We have prior terrain information
- Unexplored Areas: Expect Features to be present
- Assumption: Density of features is approximately equal to that seen in the prior terrain features
- Expected Feature Locations: Randomly distributed in the unexplored area
- We now use these expected locations to predict localisation performance
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A* Planning Algorithm

Get Initial Terrain Info.
Get Expected Terrain Info.

Destination Reached?

Yes

Finished Planning, Fly Path

No

Goto End of Local Path

From Current Location, get Local Waypoint Options,
Order Options based on Cost of Distance to Destination

Check Localisation Error and Time Constraints

Meets Constraints?

Yes

Exhausted All Local Options?

No

Next Option

Go Back to Start of Local Path
A* Planning Algorithm

Discarded Trajectories
(Violated Localisation Error Constraints)

Current Point of Planning

Planned Trajectory

Local Waypoints

Easting (m)

Northing (m)
Example Terrain Set 1
Example Terrain Set 1

Expected Uncertainty Values for Planned Trajectory

- Yaw (deg)
- Roll/Pitch (deg)
- Hor. Pos. (m)
- Vert. Pos. (m)

Time (secs)
Example Terrain Set 2
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Conclusions

• **Active SLAM**
  - Studies the connection between localisation, mapping and control
  - Develop intelligent guidance strategies to maximise SLAM performance

• **Issues in Active SLAM**
  - Computational complexity, utility calculation and action space
  - Uncertainty in Approximated Utility
  - Utility of exploration vs. revisiting
  - Multi-objective Missions