Signals and Modulation

The Nature of Electronic Signals

- Signals are generally classified in terms of their time or frequency domain behaviour
- Time domain classifications include:
  - Static: Unchanging (DC)
  - Quasi static: Slowly changing (amplifier drift)
  - Periodic: Sine wave
  - Repetitive: Periodic but varying (Electrocardiograph)
  - Transient: Occur once only (impulse)
  - Quasi transient: Repetitive but seldom (radar pulses)

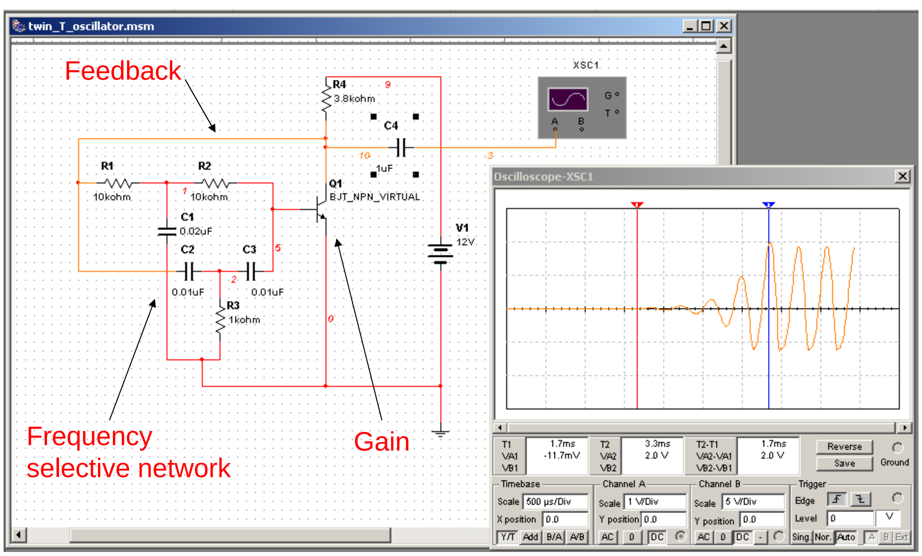
Sinusoidal Signals

- Most acoustic and electromagnetic sensors exploit the properties of sinusoidal signals
- In the time domain, such signals are constructed of sinusoidally varying voltages or currents constrained within wires

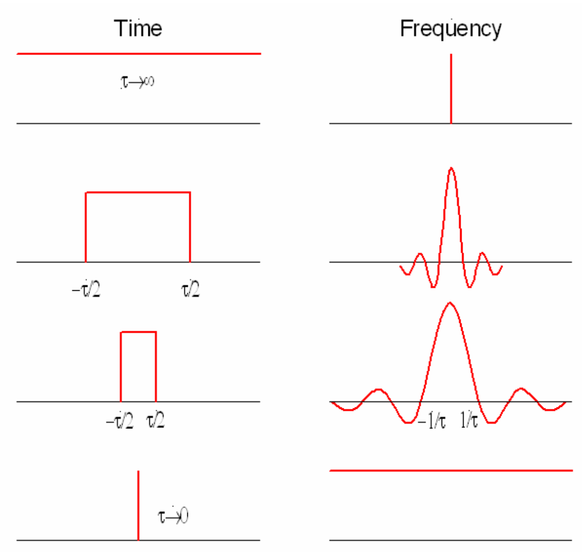
$$v_c(t) = A_c \cos \omega_c t = A_c \cos 2\pi f_c t$$

where $v_c(t)$ – Signal
A_c – Signal amplitude (V)
ω_c – Frequency (rad/s)
f_c – Frequency (hz)
t – Time (s)

Generating Sinusoidal Signals



Sinusoidal Signals in the Frequency Domain



The accuracy with which the frequency of a signal can be determined is inversely proportional to the observation time

The Fourier Series

- All continuous periodic signals can be represented by a fundamental frequency sine wave and a collection of sine and/or cosine harmonics of that fundamental sine wave
- The Fourier series for any waveform can be expressed in the following form:

$$v(t) = \frac{a_0}{2} + \int_{n=1}^{\infty} a_n \cos(n\omega t) + \int_{n=1}^{\infty} b_n \sin(n\omega t)$$

where: a_n b_n Amplitudes of the harmonics (can be zero)

n Integer

Calculating the Fourier Coefficients

- The amplitude coefficients can be calculated as follows

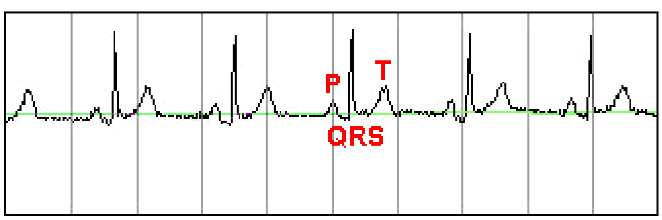
$$a_n = \frac{2}{T}\int_0^t v(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^t v(t)\sin(n\omega t)dt$$

- Because only certain frequencies, determined by integer *n*, are allowed, the spectrum is discrete
- The term $a_0/2$ is the average value of $v(t)$ over a complete cycle
- Though the harmonic series is infinite, the coefficients become so small that their contribution is considered to be negligible.

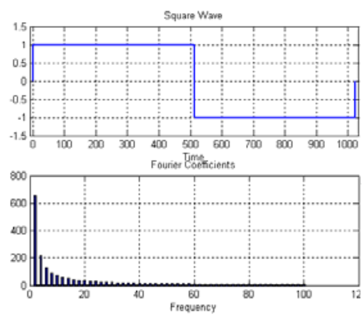
Number of Harmonics Required

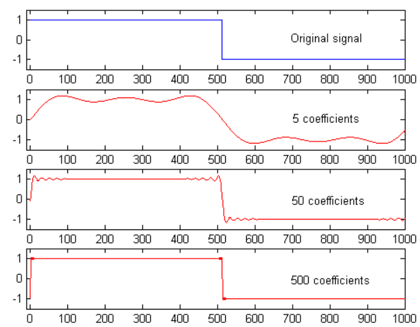
- An ECG trace, with a fundamental frequency of about 1.2Hz can be reproduced with 70 to 80 harmonics (a bandwidth of about 100Hz)



- A square wave on the other hand may require up to 1000 harmonics, because extremely high frequency terms are contained within the transitions

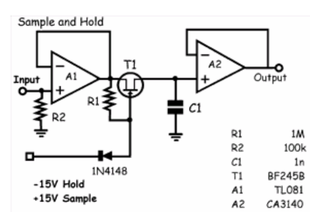
Harmonics Required for Square Wave



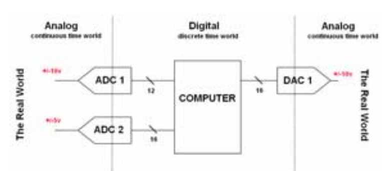


Sampled Signals and Digitisation

- To process signals within a computer requires that they be sampled periodically and then converted to a digital representation
- To ensure accurate reconstruction:
  - the signal must be sampled at a rate which is at least double the highest significant frequency component of the signal. This is known as the Nyquist rate.
  - The number of discrete levels to which the signal is quantised must also be sufficient.
- Signal reconstruction involves holding the signal constant (zero order hold) during the period between samples then passing the signal through low-pass filter to remove high frequency components generated by the sampling process

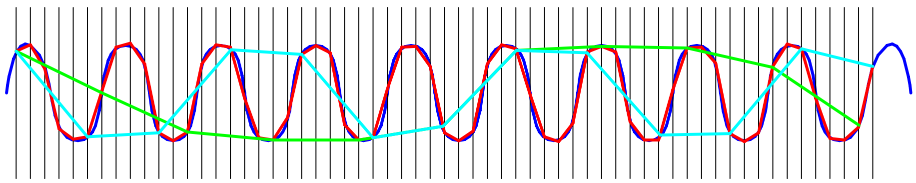


Sample & Hold



Digital Signal Processor

Aliasing: Time Domain Illustration

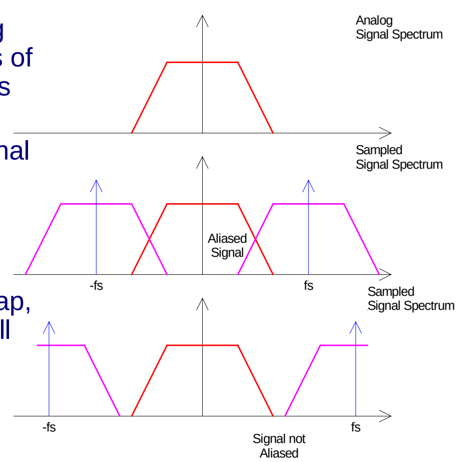


- Original Sine wave
- Sampled 6.66x per cycle
- Sampled 1.33x per cycle
- Sampled 1.11x per cycle

- Data sampled at less than the Nyquist rate appears to be shifted in frequency

Aliasing: Frequency Domain Illustration

- In the frequency domain, an analog signal may be represented in terms of its amplitude and total bandwidth as shown in the figure.
- A sampled version of the same signal can be represented by a repeated sequence spaced at the sample frequency (generally denoted fs)
- If the sample rate is not sufficiently high, then the sequences will overlap, and high frequency components will appear at a lower frequency (albeit possibly with reduced amplitude)



Chirp to 500Hz sampled at 1kHz

Sampled at 500Hz

Sampled at 250Hz

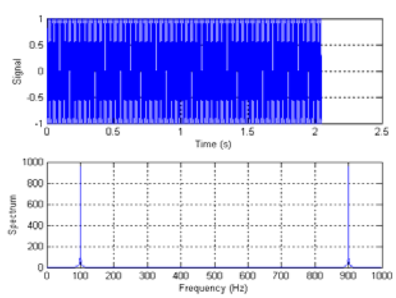
Generating Signals in MATLAB

- MATLAB has a comprehensive number of built-in functions to facilitate the generation of both periodic and aperiodic functions
  - Square – Generates a square wave
  - Sawtooth – Generates a triangular wave
  - Sine – Generates a sine wave

  - Exp – Generates an exponential

Using the FFT in MATLAB

- MATLAB has a good Fast Fourier Transform (FFT) function, but be sure to remember toe following
  - Select a signal of length 2^n
  - For a real input the output spectrum is folded
  - The spectrum is complex, so use the “abs”




```
dt=1.0e-03;                    % sample period (s)
t=(1:2048)*dt;                 % total time (n=11)
f=100;                         % signal frequency (Hz)
sig=cos(2*pi*f.*t);            % generate time domain sig
subplot(211), plot(t,sig);     % plot time domain signal
spect = abs(fft(sig));         % FFT to obtain spectrum
freq=(0:2047)./(dt*2048);      % generate the freq axes
subplot(212), plot(freq,spect); % plot the spectrum
```

Filtering

Filter Types

- A filter is a frequency selective network that passes certain frequencies of an input signal and attenuates others
- The three common types of filter are:
  - High Pass
  - Low Pass
  - Band Pass
- High pass filter blocks signals below its cutoff frequency and passes those above
- Low pass filter passes signals below its cutoff frequency and attenuates those above
- Band pass filter passes a range of frequencies while attenuating those both above and below that range
- A fourth less common configuration is a band-stop or notch filter that attenuates signals at a specific frequency or over a narrow range of frequencies and passes all other frequencies.

Filter Implementations

- The major filter categories are as follows
  - Butterworth (maximally flat)
  - Chebyshev (equi ripple)
  - Bessel (linear phase)
  - Elliptical
- Note that the pass band is specified by its half power points (0.707 of the peak voltage gain)
- If the gain is plotted in dB then it would be calculated using 20*log10(gain) to convert to power.

Butterworth

- This approximation to an ideal low pass filter is based on the assumption that a flat response at zero frequency is most important.
- The transfer function is an all-pole type with roots that fall on the unit circle
- Fairly good amplitude and transient characteristics

Chebyshev

- The transfer function is also all-pole, but with roots that fall on an ellipse. This results in a series of equi amplitude ripples in the pass band and a sharper cutoff than Butterworth.
- Good selectivity but poor transient behaviour

Bessel

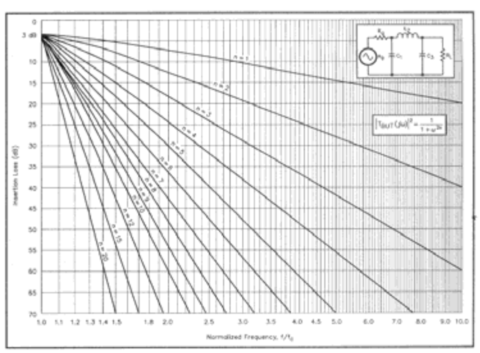
- Optimised to obtain a linear phase response which results in a step response with no overshoot or ringing and an impulse response with no oscillatory behaviour
- Poor frequency selectivity compared to the other response types

Elliptic

- Has zeros as well as poles which create equi-ripple behaviour in the pass band similar to Chebyshev filters
- Zeros in the stop band reduce the transition region so that extremely sharp roll-off characteristics can be achieved

Filter Rolloff and Insertion Loss

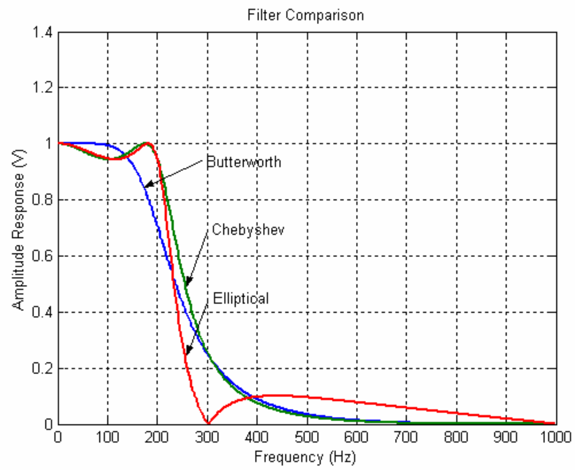
- The rate at which a signal is attenuated as a function of frequency is known as rolloff
- The total attenuation at a specific frequency is know as the insertion loss
- Insertion loss at a particular frequency is determined by the filter order, n
- Rolloff for lowpass filters asymptotic to 6n dB/octave
- Rolloff for bandpass filters asymptotic to 3n dB/octave



Stop-band loss of Butterworth low-pass filters. The almost vertical angle of the lines representing filters with high values of n (10, 12, 15, 20) show the slope of the filter will be very high (sharp cutoff).

Note – 1 octave = doubling in frequency
1 decade = 10 x frequency

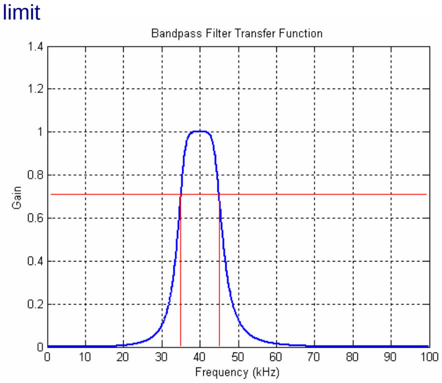
Filter Frequency Responses



Note that the cutoff frequency (200Hz in this case) specified in MATLAB is equal to the 3dB point for the Butterworth filter and to the passband ripple for the Chebyshev and Elliptic filters

Specifying a Butterworth Bandpass Filter

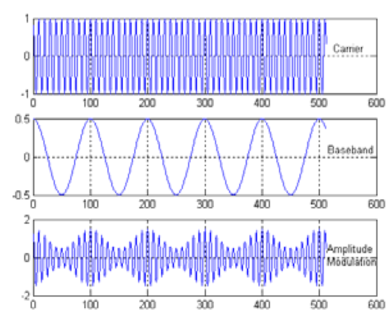
```
% Banspass filter
fs = 200e+03;
ts = 1/fs;
fmat = 40.0e+03;            % centre frequency
bmat = 10.0e+03;            % bandwidth
wl=2*ts*(fmat-bmat/2);      % lower band limit
wh=2*ts*(fmat+bmat/2);      % upper band limit
wn=[wl,wh];
[b,a]=butter(3,wn);         % 6th order
[h,w]=freqz(b,a,1024);
freq=(0:1023)/(2000*ts*1024);
%semilogx(freq,20*log10(abs(h)));
plot(freq,abs(h));
grid
title('Bandpass Filter Transfer Function')
xlabel('Frequency (kHz)');
ylabel('Gain')
```



Modulation

Amplitude Modulation (AM)

- A modulation technique in which the amplitude of the carrier is varied in accordance with some characteristic of the baseband modulating signal.
- It is the most common form of modulation because of the ease with which the baseband signal can be recovered from the transmitted signal



$$v_{am}(t) = A_c[1 + v_b(t)]\cos 2\pi f_c t$$

Percentage Modulation

$$v_{am}(t) = A_c\left[1 + A_{am}\cos 2\pi f_a t\right]\cos 2\pi f_c t$$

- In general A_{am}<1 otherwise a phase reversal occurs and demodulation becomes more difficult.
- The extent to which the carrier has been amplitude modulated is expressed in terms of a *percentage modulation* which is just calculated by multiplying A_{am} by 100.

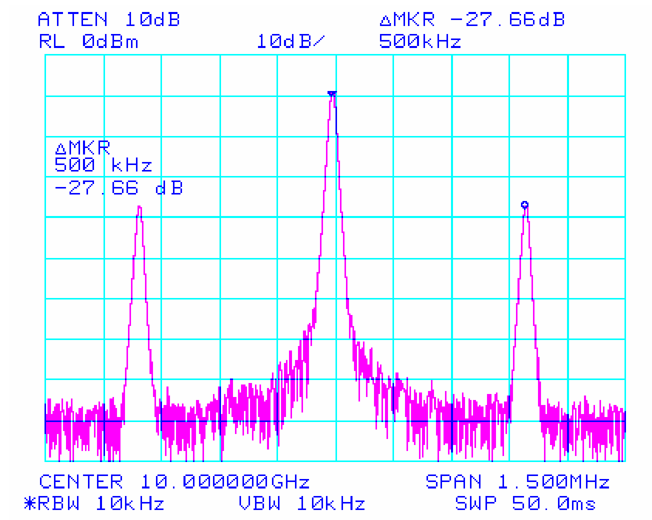
AM in the Frequency Domain

- To determine the characteristics of the signal in the frequency domain, it can be rewritten in the following form (using a trig identity)

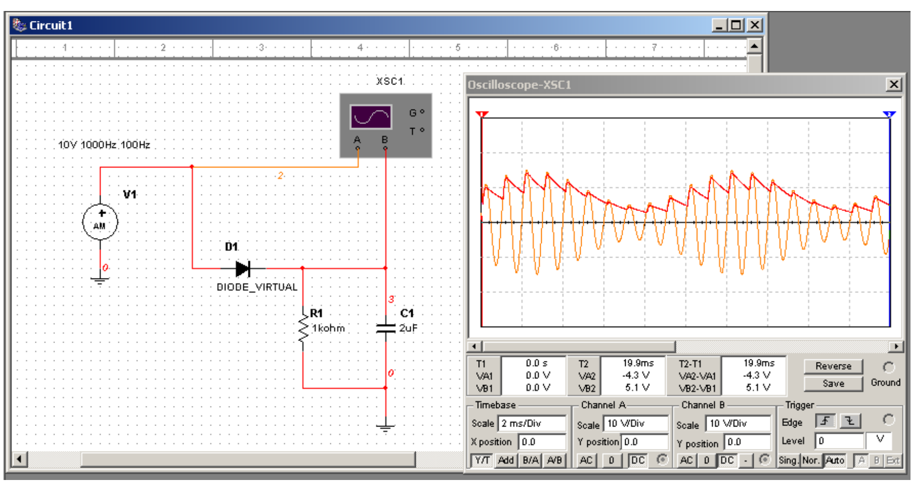
$$v_{am}(t) = A_c\cos 2\pi f_c t + \frac{A_c A_{am}}{2}\left[\cos 2\pi(f_c - f_a)t + \cos 2\pi(f_c + f_a)t\right]$$

- It can be seen that this is made up from three independent frequencies:
  - The original carrier
  - A frequency at the difference between the carrier and the baseband signal
  - A frequency at the sum of the carrier and the baseband signal

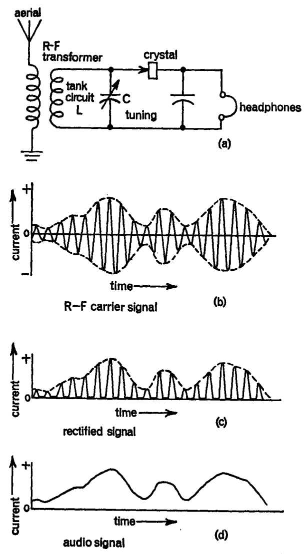
Spectral Content of Amplitude Modulation



AM Demodulation: Simulation



AM Demodulation: The Crystal Set



Frequency Modulation (FM)

- A modulation technique in which the frequency of the carrier is varied in accordance with some characteristic of the baseband signal.

$$v_{fm}(t) = A_c \cos\left[\omega_c t + k \int_{-\infty}^{t} v_b(t)dt\right]$$

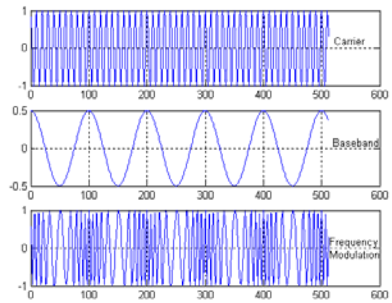
- The modulating signal is integrated because variations in the modulating term equate to variations in the carrier phase.
- The instantaneous angular frequency can be obtained by differentiating the instantaneous phase as shown

$$\omega = \frac{d}{dt}\left[\omega_c t + k \int_{-\infty}^{t} v_b(t)dt\right] = \omega_c + k v_b(t)$$

Sinusoidal FM Modulation

- For sinusoidal modulation, the formula for FM is as follows:

$$v_{fm}(t) = A_c \cos[\omega_c t + \beta \sin \omega_a t]$$



- In this case β which is the maximum phase deviation is usually referred to as the *modulation index*
- The instantaneous frequency in this case is

$$f = \frac{\omega_c}{2\pi} + \frac{\beta\omega_a}{2\pi}\cos\omega_a t$$

$$f = f_c + \beta f_a \cos\omega_a t$$

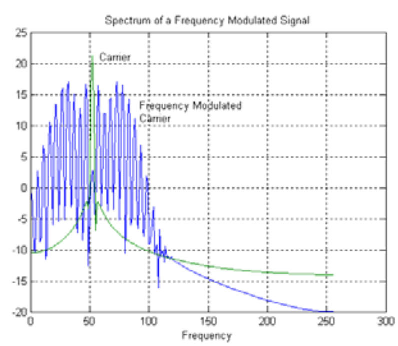
- So the maximum frequency deviation defined as Δf

$$\Delta f = \beta f_a$$

FM Spectrum

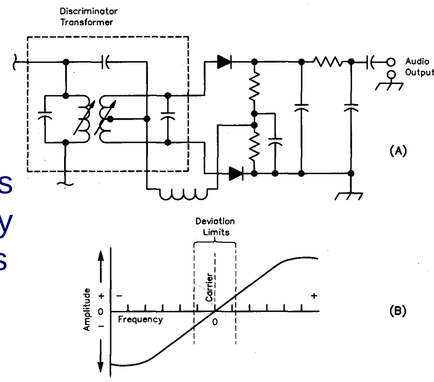
- Even though the instantaneous frequency lies within the range f_c+/-Δf, the spectral components of the signal don't lie within this range
- The spectrum comprises a carrier with amplitude $J_o(\beta)$ with sidebands on either side of the carrier at offsets of ω_a, $2\omega_a$, $3\omega_a$, ….
- The bandwidth is infinite, however, for any β, most of the power is confined within finite limits
- Carson's Rule states that the bandwidth is about twice the sum of the maximum frequency deviation plus the modulating frequency

$$BW \approx 2(\Delta f + f_a)$$



FM Demodulation

- Demodulation of FM is commonly achieved by converting to AM followed by envelope detection
- Simplest conversion is to pass the signal through a frequency sensitive circuit like a lowpass or bandpass filter – called a discriminator
- Modern methods include PLL and quadrature detection



Linear Frequency Modulation

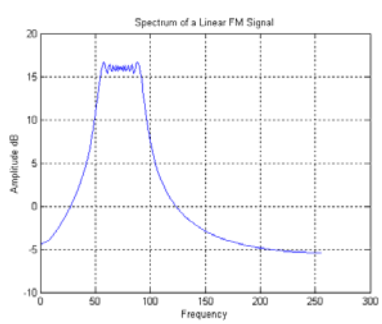
- In most active sensors, the frequency is modulated in a linear manner with time

$$\omega_b = A_b t$$

- Substituting into the standard equation for FM, we obtain the following result

$$v_{fm}(t) = A_c \cos\left[\omega_c t + A_b \int_{-\infty}^{t} t\,dt\right]$$

$$v_{fm}(t) = A_c \cos\left[\omega_c t + \frac{A_b}{2} t^2\right]$$



Frequency Modulated Continuous Wave Processing

- In Frequency Modulated Continuous Wave (FMCW) systems, a portion of the transmitted signal is mixed with (multiplied by) the returned echo.
- The transmit signal will be shifted from that of the received signal because of the round trip time τ

$$v_{fm}(t-\tau) = A_c \cos\left[\omega_c(t-\tau) + \frac{A_b}{2}(t-\tau)^2\right]$$

- Calculating the product

$$v_{fm}(t-\tau)v_{fm}(t) = A_c^2 \cos\left[\omega_c t + \frac{A_b}{2}t^2\right]\cos\left[\omega_c(t-\tau) + \frac{A_b}{2}(t-\tau)^2\right]$$

FMCW continued…..

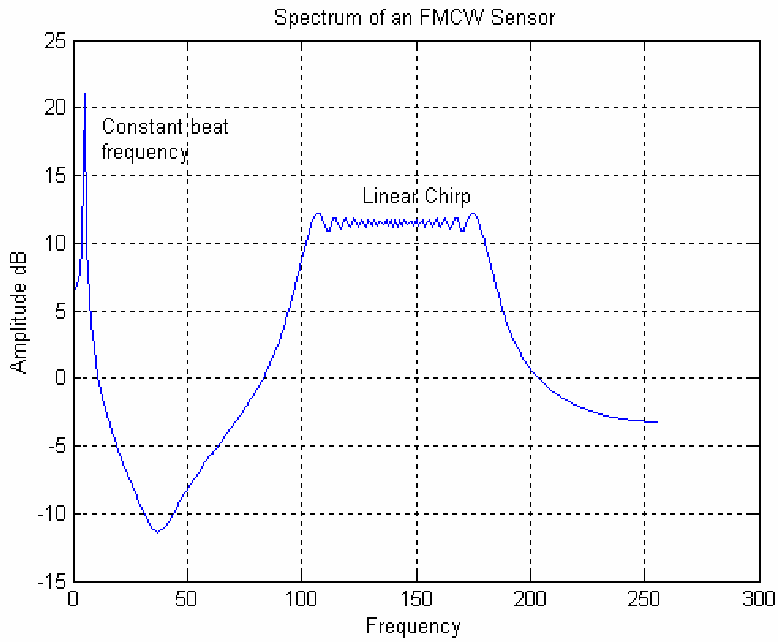
- Equating using the trig identity for the product of two sines cos*A*.cos*B*=0.5[cos(*A*+*B*)+cos(*A*-*B*)]

$$v_{out}(t) = \frac{1}{2}\left[\begin{array}{l}\cos\left\{(2\omega_c - A_b\tau)t + A_b t^2 + \left(\frac{A_b}{2}\tau^2 - \omega_c\tau\right)\right\} \\ +\cos\left\{A_b\tau.t + \left(\omega_c\tau - \frac{A_b}{2}\tau^2\right)\right\}\end{array}\right]$$

- The first cos term describes a linearly increasing FM signal (chirp) at about twice the carrier frequency. This term is generally filtered out.
- The second cos term describes a beat signal at a fixed frequency

$$f_{beat} = \frac{A_b}{2\pi}\tau$$

- The signal frequency is directly proportional to the delay time τ, and hence is directly proportional to the round trip time to the target

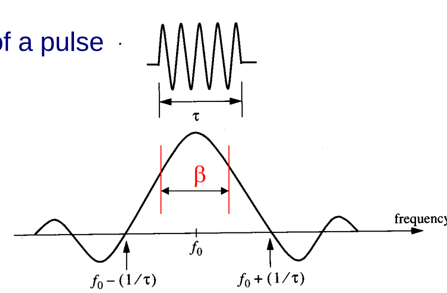


Pulse Amplitude Modulation

- Modulation in which the amplitude of individual, regularly spaced pulses in a pulse train is varied in accordance with some characteristic of the modulating signal
- For time-of-flight sensors, the amplitude is generally constant
- The ability of a pulsed sensor to resolve two closely spaced targets is determined by the pulse width
- The 3dB (50% power) bandwidth of a pulse is

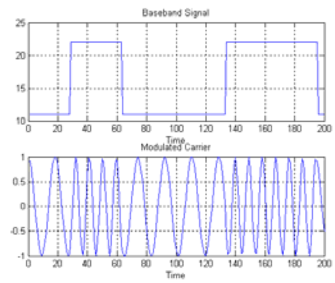
$$\beta \approx \frac{1}{\tau}$$

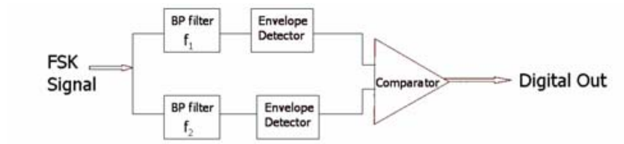
where β - Spectral Width (Hz)
τ - Pulsewidth (sec)



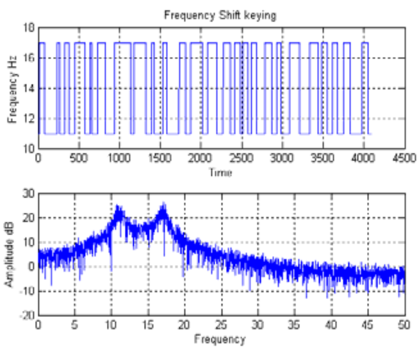
Frequency Shift Keying (FSK)

- This modulation technique is the digital equivalent of linear FM where only two different frequencies are utilised
- A single bit can be represented by a single cycle of the carrier, but if the data rate is not critical, then multiple cycles can be used
- Demodulation can be achieved by detecting the outputs of a pair of filters centred at the two modulation frequencies

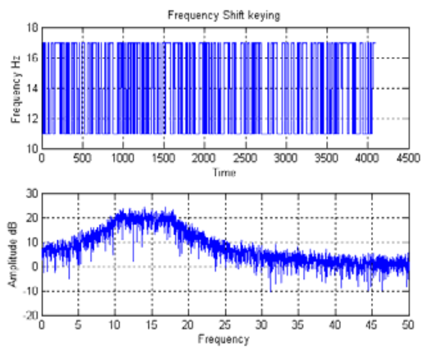




Effect of the Number of Cycles per Bit on the Signal Spectrum



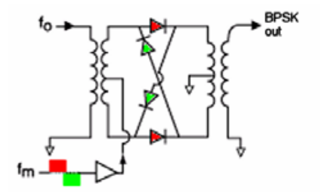
5 Cycles per Bit

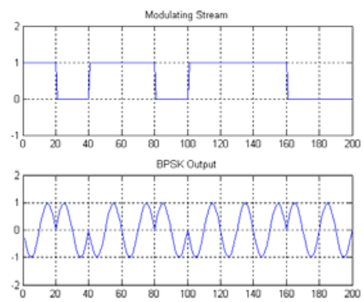


One Cycle per Bit

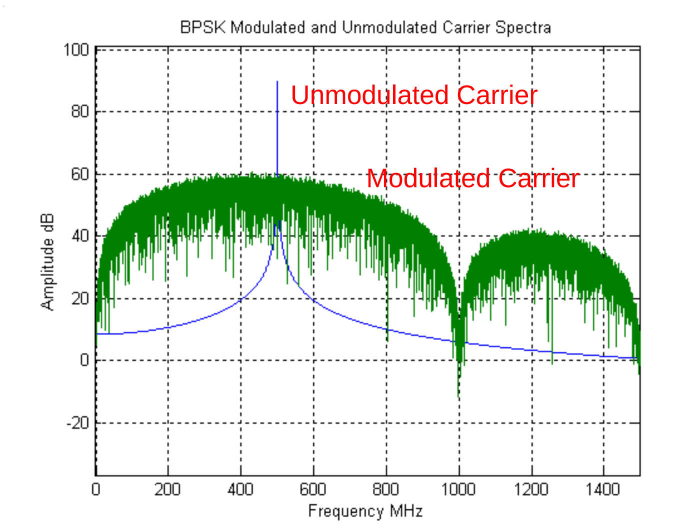
Phase Shift Keying (PSK)

- Usually binary phase coding. The carrier is switched between +/-180° according to a digital baseband sequence.
- This modulation technique can be implemented quite easily using a balanced mixer shown, or with a dedicated BPSK modulator
- Demodulation is achieved by multiplying the modulated signal by a coherent carrier (a carrier that is identical in frequency and phase to the carrier that originally modulated the BPSK signal).
- This produces the original BPSK signal plus a signal at twice the carrier which can be filtered out.



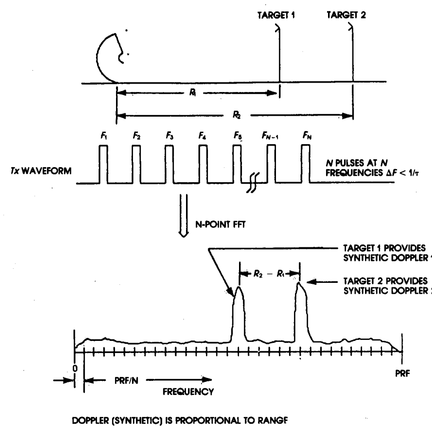


Spectrum of a BPSK Signal
One Cycle per Bit Random Modulation



Stepped Frequency Modulation

- A sequence of pulses are transmitted each at a slightly different frequency
- The pulse width (in the radar case) is made sufficiently wide to span the region of interest, but because it is so wide, it cannot resolve individual targets within that region
- If all of the pulses are processed together, the effective resolution is improved because the total bandwidth is widened by the total frequency deviation of the sequence of pulses.



Convolution

Linear Time Invariant (LTI) Systems

- LTI systems are completely characterised by their impulse responses
- Applying an impulse $\delta(t)$ to the input of an LTI system is therefore a good method of determining its characteristics
- An arbitrary input can be expressed as a weighted superposition of time shifted impulses
- Therefore the system output is just the weighted superposition of the time shifted impulse responses
- This weighted superposition is termed the convolution sum (discrete time) or convolution integral (continuous time)

Convolution Integral

- The relationship between the output $y(t)$ and the input $x(t)$ of an LTI system is given by the convolution with the impulse response

$$y(t) = \int_{\tau=-\infty}^{t} h(t-\tau)x(\tau)d\tau = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

- If the Laplace transform is taken of this integral

$$Y(s) = H(s)X(s)$$

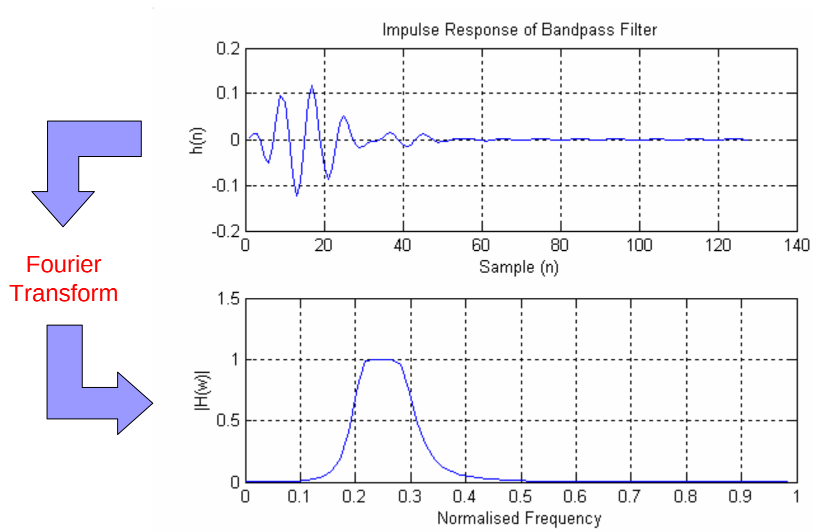
- That is, convolution in the time domain is equivalent to multiplication in the Laplace domain

Convolution and Frequency Response

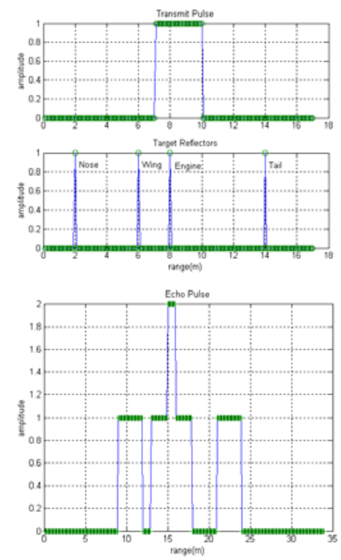
- If *s* is replaced by its imaginary component, $j\omega$, then

$$Y(\omega) = H(\omega)X(\omega)$$

- Which describes the frequency response of the system
- Therefore, $|H(\omega)|$ is the transfer function that characterises the filter in the frequency domain, and it is this representation that we discussed in the section on filters
- It is now the Fourier transform that maps from the time domain to the frequency domain



Convolution: Time of Flight



- For time-of-flight sensors the impulse response of the round-trip propagation to the point target is a signal that is delayed in time and attenuated in amplitude $h(t) = a\delta(\tau-\beta)$ where a represents the attenuation and β the round trip time delay
- This is an LTI system which allows us to calculate the system response by taking the convolution of the transmit pulse with the target (made of many point reflectors)