

Chapter 14.

Doppler Measurement

14.1. The Doppler Shift

The apparent difference between the frequency at which sound or light waves leave a source and that at which they reach an observer, caused by relative motion of the observer and the wave source. This phenomenon is used in astronomical measurements, in radar and modern navigation. It was first described (1842) by Austrian physicist Christian Doppler.

Examples of the Doppler effect include the following:

- As one approaches a blowing horn, the perceived pitch is higher until the horn is reached and then becomes lower as the horn is passed.
- The light from a star, observed from the Earth, shifts toward the red end of the spectrum (lower frequency or longer wavelength) if the Earth and star are receding from each other and toward the violet (higher frequency or shorter wavelength) if they are approaching each other. The Doppler effect is used in studying the motion of stars.

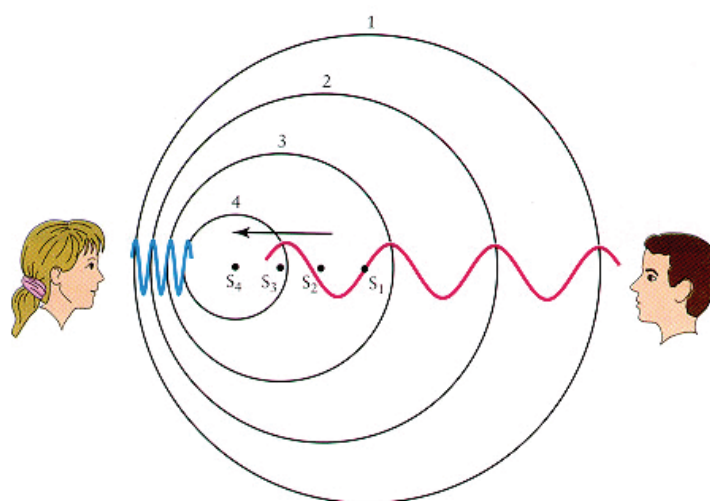


Figure 14.1: Illustration of Doppler shift for a sound source moving from S_1 to S_4

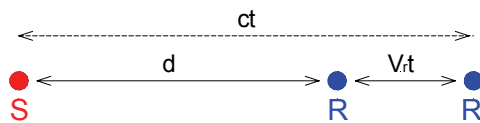
14.1.1. Doppler Shift Derivation

Consider the relationship between the frequency of sound produced by a source moving with velocity v_s and the frequency received by a receiver moving with velocity v_r . For simplicity, assume that the both the source and the receiver are moving in a straight line in the same direction.

At time $t = 0$, the source, S, and receiver R are separated by a distance d .



The source emits a wave that propagates at velocity c and reaches the receiver after time t .



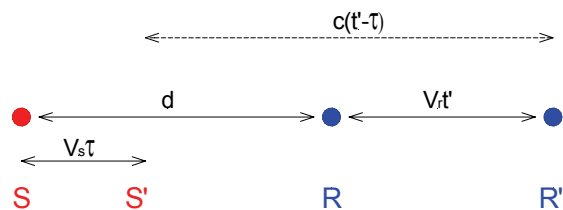
As the receiver has moved $v_r t$ metres

$$ct = d + v_r t, \quad (14.1)$$

or

$$t = \frac{d}{c - v_r}. \quad (14.2)$$

At time τ the source, S, would have moved $v_s \tau$. Let the wave emitted at that instant be received at time t' by R. In this time R would have moved $v_r t'$



$$c(t' - \tau) = (d - v_s \tau) + v_r t', \quad (14.3)$$

making

$$t' = \frac{d + (c - v_s) \tau}{c - v_r}, \quad (14.4)$$

Thus for the receiver, the interval between the waves has been

$$\tau' = t' - t = \frac{c - v_s}{c - v_r} \tau. \quad (14.5)$$

Whereas for the source, the interval between waves has been τ . Now the number of waves emitted in τ by the source must equal the number of waves received by the receiver in τ'

$$f_r \tau' = f_s \tau, \quad (14.6)$$

making

$$f_r = \frac{c - v_r}{c - v_s} f_s. \quad (14.7)$$

For v_s and $v_r \ll c$

$$f_r = \frac{1 - v_r/c}{1 - v_s/c} f_s = \left[1 - \frac{v_r}{c} \right] \left[1 - \frac{v_s}{c} \right]^{-1}. \quad (14.8)$$

Expanding the last term using the binomial expansion

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots \quad (14.9)$$

For $x \ll 1$, the higher order terms can be ignored

$$f_r \approx \left[1 - \frac{v_r}{c} \right] \left[1 + \frac{v_s}{c} \right] = \left(1 - \frac{v_{rs}}{c} \right) f_s. \quad (14.10)$$

where $v_{rs} = v_r - v_s$ is the velocity of the receiver relative to the source

The Doppler shift is thus

$$f_d = f_r - f_s = \frac{-v_{rs}}{c} f_s. \quad (14.11)$$

The frequency moving away from the source will be less than the frequency measured at the source, whereas the frequency measured at a receiver moving towards the source will be greater than the frequency measured at the source

14.2. Doppler Geometry

In most Doppler sensors both the transmitter and receiver are stationary, and they illuminate a moving target. In addition, for some applications (Doppler ultrasound imaging) they may not be co located as shown below

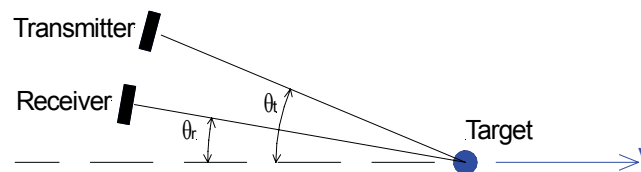


Figure 14.2: Doppler geometry: separated transducers

The velocity of the target relative to the transmitter will be $v \cos \theta_t$ and the velocity of the target relative to the receiver will be $v \cos \theta_r$.

The Doppler shift arising under these circumstances can be calculated assuming that:

- The target is a receiver moving away from the source with a velocity $v \cos \theta_t$
- The receiver is moving away from the target (source) with a velocity $v \cos \theta_r$.

This is equivalent to the receiver moving away from the source with velocity $v \cos \theta_t + v \cos \theta_r$ even though both are stationary.

14.2.1. Targets moving at low velocities ($v \ll c$)

The Doppler frequency for separated transducers is

$$f_d = -\frac{f_s v}{c} (\cos \theta_t + \cos \theta_r), \quad (14.12)$$

$$f_d = -\frac{2f_s v}{c} \cos\left(\frac{\theta_r + \theta_t}{2}\right) \cos\left(\frac{\theta_r - \theta_t}{2}\right). \quad (14.13)$$

If the transmit and receiver transducers are co located then $\theta_r \approx \theta_t = \theta$ and the formula for the Doppler frequency reduces to

$$f_d = -\frac{2f_s v}{c} \cos \theta = -\frac{2v}{\lambda_s} \cos \theta. \quad (14.14)$$

14.2.2. Targets Moving at High Speed ($v < c$)

In this case it is not possible to use the EM radiation approximation, and the whole formula must be used for separated transducers. This is almost always the case with ultrasound measurements in air.

$$f_r = \frac{c - v \cos \theta_r}{c + v \cos \theta_t} f_s, \quad (14.15)$$

And the Doppler frequency is

$$f_d = f_r - f_s. \quad (14.16)$$

For co-located transducers $\theta_r \approx \theta_t = \theta$ just substitute for a common angle.

14.3. Doppler Frequency Extraction

This section outlines the instrumentation required to detect Doppler shifts in a received signal.

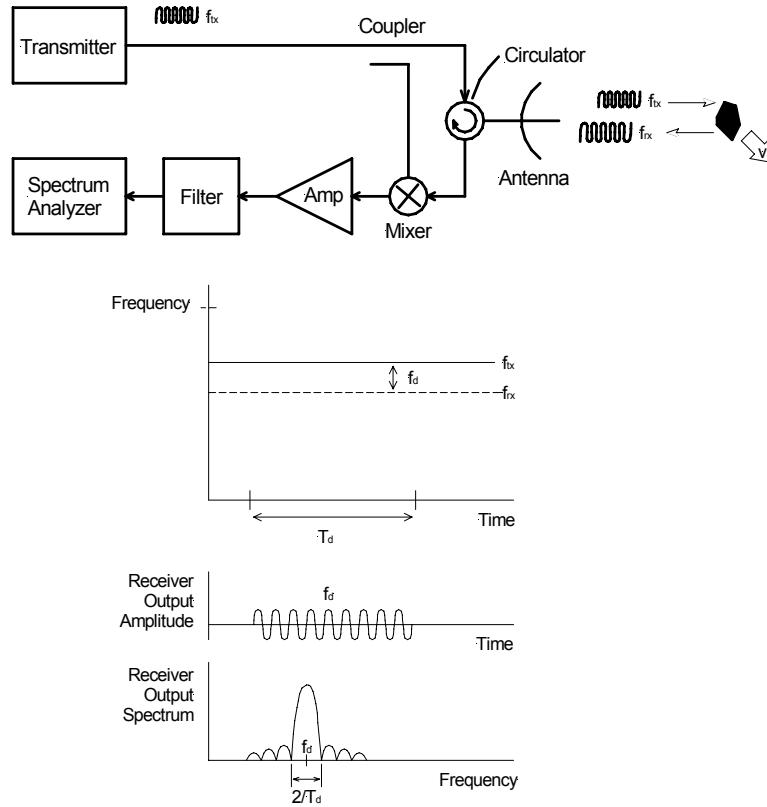


Figure 14.3: Configuration of a generic Doppler sensor

The transmitted signal is of the form

$$x_t(t) = \xi_t \cos(\omega_s t) \quad (14.17)$$

The corresponding received signal from a single target will be

$$x_r(t) = \xi_r \cos([\omega_s + \omega_d]t + \phi), \quad (14.18)$$

where ϕ - phase term dependent on the distance to the target (rad),

$$\omega_s - 2\pi f_s \text{ (rad/s),}$$

$$\omega_d - 2\pi f_d \text{ (rad/s).}$$

The two signals are mixed (multiplied together) to produce

$$\begin{aligned} x_t(t)x_r(t) &= \xi_t \xi_r \cos(\omega_s t) \cos([\omega_s + \omega_d]t + \phi) \\ &= \frac{\xi_t \xi_r}{2} \{ \cos(\omega_d t + \phi) + \cos([2\omega_s + \omega_d]t + \phi) \} \end{aligned} \quad (14.19)$$

This signal is low pass filtered as shown in the figure below to remove the component at $2f_s$ leaving only the Doppler signal

$$x_d(t) = \frac{\xi_t \xi_r}{2} \cos(\varpi_d t + \phi). \quad (14.20)$$

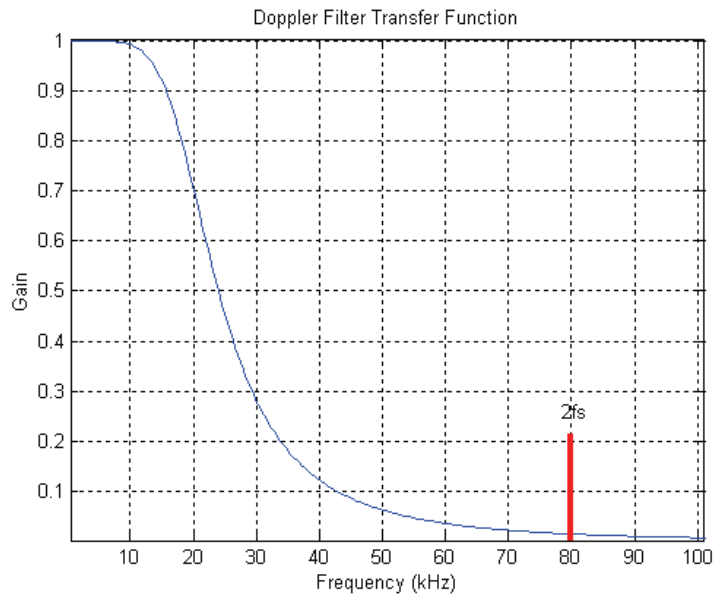


Figure 14.4: Lowpass filter to remove the component at $2f_s$

The reflected signal amplitude from non-moving objects in the beam will be 40 to 50dB larger than the Doppler signal, and so additional high pass filtering is often required to remove this.

14.3.1. Direction Discrimination

The Doppler process discussed above can provide only an absolute difference frequency, it contains no information regarding the direction of motion.

A number of techniques can be applied to preserve this directional information:

- Sideband filtering
- Offset carrier demodulation
- In-phase/ quadrature demodulation

In the descriptions that follow, it must be remembered that

- $\varpi_d > 0$ Target velocity towards the sensor
- $\varpi_d < 0$ Target velocity away from the sensor

Sideband Filtering

The received signal is split and passed through two bandpass filters, one passing signals over the range $\omega_s < \omega < \omega_s + \omega_m$ and the other passing signals over the range $\omega_s - \omega_m < \omega < \omega_s$. The output of each filter passes through a mixer and filter as usual.

If the target is approaching the signal appears in the first bin, and if it is receding then it appears in the second

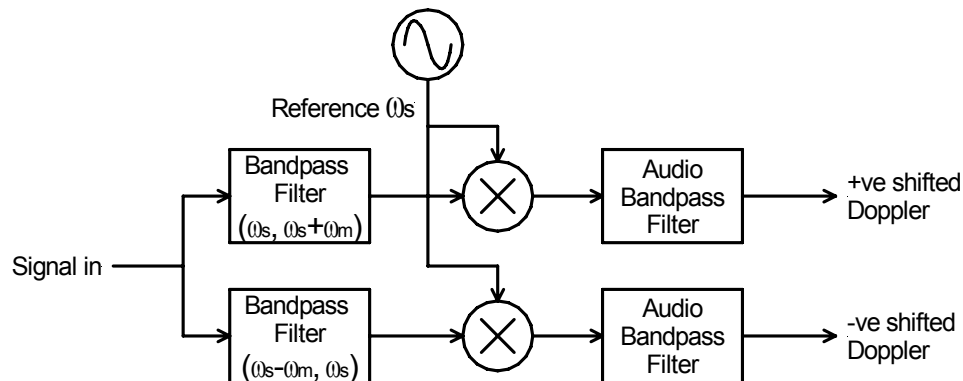


Figure 14.5: Sideband filtering

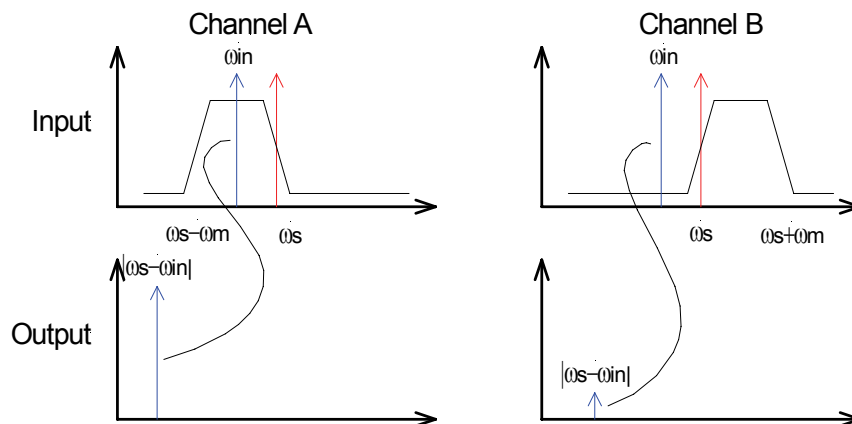


Figure 14.6: Sideband filtering spectra in the two channels

Offset Carrier Demodulation

This process involves heterodyning (mixing) the received signal by a reference signal $\omega_1 + \omega_s$.

The received signal is

$$x_r(t) = \xi_r \cos([\omega_s + \omega_d]t + \phi), \quad (14.21)$$

and the reference signal is

$$x_1(t) = \xi_1 \cos([\omega_s + \omega_1]t). \quad (14.22)$$

Mixing the two signals (14.21) and (14.22) gives

$$x_1(t)x_r(t) = \frac{\xi_1 \xi_r}{2} \{ \cos([\omega_1 + \omega_d]t + \phi) \cos([2\omega_s + \omega_1 + \omega_d]t + \phi) \}, \quad (14.23)$$

where ω_1 is chosen so that $\omega_1 > |\omega_{d \max}|$,

and as usual the mixed signal is filtered to remove the component at $2\omega_s$.

$\omega_1 + \omega_d > \omega_1$ +ve shift Doppler

$\omega_1 + \omega_d < \omega_1$ -ve shift Doppler

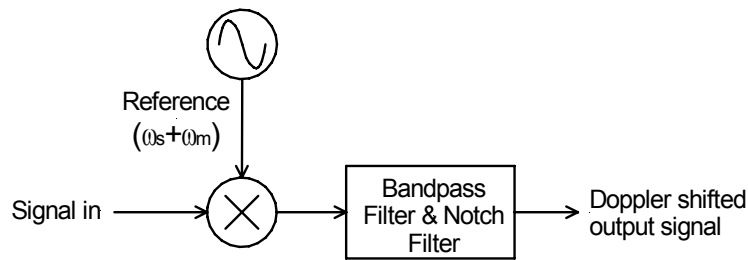


Figure 14.7: Offset carrier demodulation

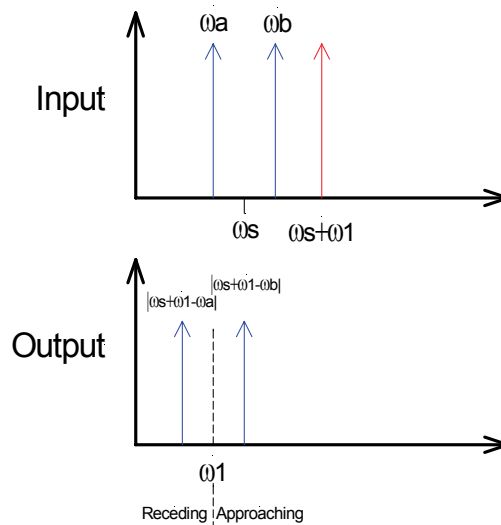


Figure 14.8: Offset carrier spectra

In Phase/ Quadrature Demodulation

The received signal is split into two channels. In the in-phase channel, it is mixed with the transmitted signal, and in the quadrature channel it is mixed with the transmitted signal but phase shifted by $\pi/2$.

Mixing and filtering as before results in the following signals

$$\begin{aligned} i(t) &= \cos(\omega_d t + \phi) \\ q(t) &= \sin(\omega_d t + \phi) \end{aligned} \quad (14.24)$$

The direction of the Doppler shift, and hence the direction of flow is determined by noting the phase relationship between $i(t)$ and $q(t)$.

- $\omega_d > 0$, $q(t)$ is $\pi/2$ retarded with respect to $i(t)$
- $\omega_d < 0$, $q(t)$ is $\pi/2$ advanced with respect to $i(t)$

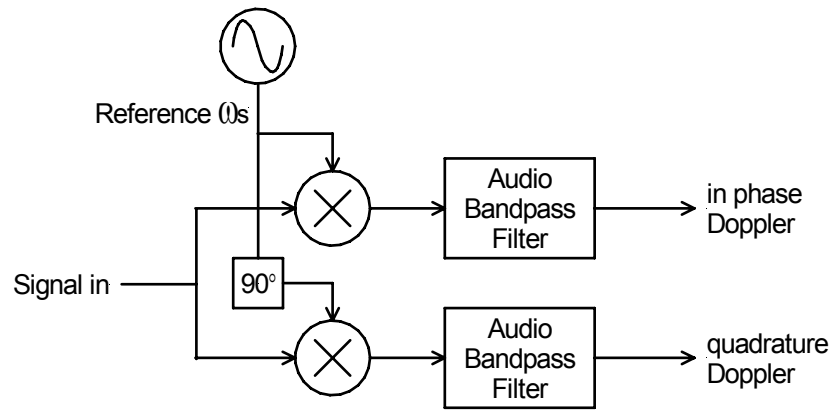


Figure 14.9: In-phase / quadrature demodulation

14.4. Pulsed Doppler

Pulsed Doppler is identical to the CW version except that the transmit signal is pulsed. This allows the technique to measure range as well as velocity. As shown in the figure, a reference version of the transmitted signal must be maintained so that the received signal can be synchronously detected

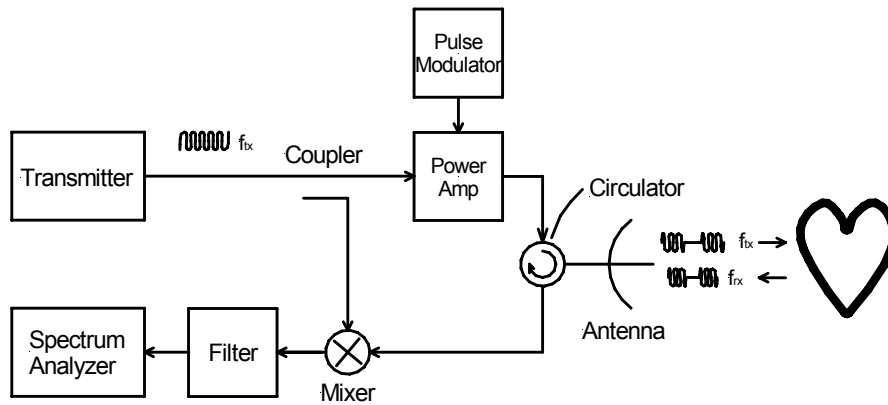


Figure 14.10: Pulsed Doppler ultrasound schematic diagram

The waveform that is received from a pulsed Doppler sensor in a specific range gate is shown in the figure below in which it should be noted that the number of cycles received during each return is a function of the pulsewidth and the Doppler frequency.

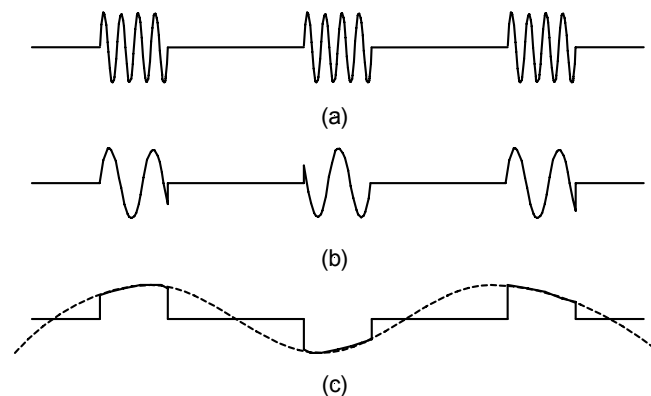
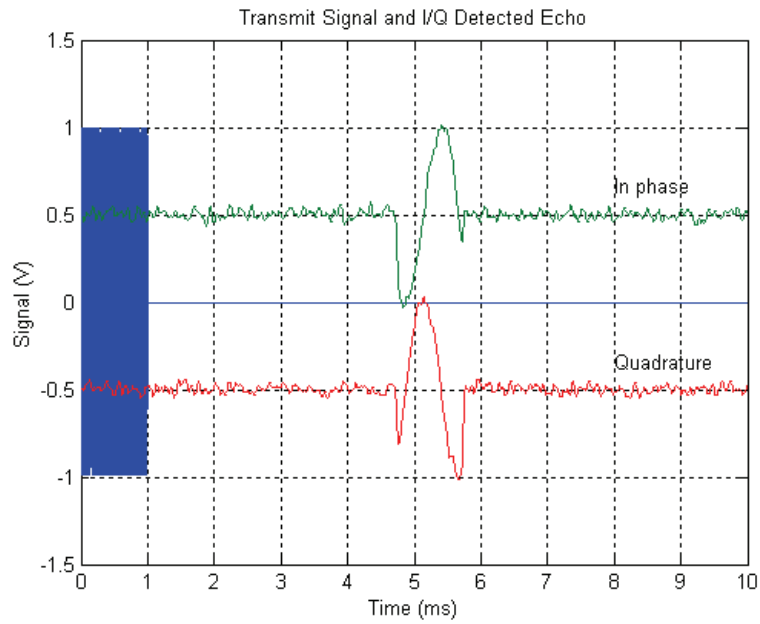


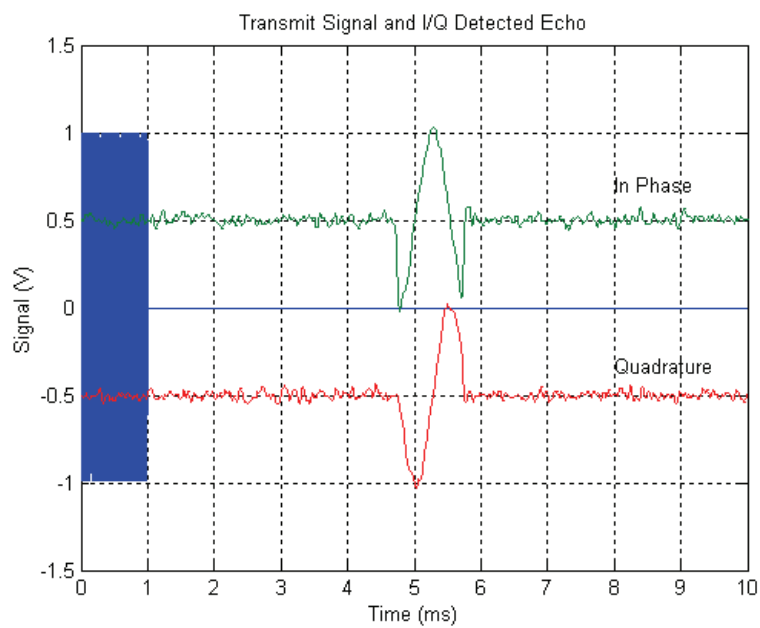
Figure 14.11: Pulsed Doppler waveforms (a) RF echo pulse train, (b) video pulse train for $f_d > 1/\tau$ and (c) video pulse train for $f_d < 1/\tau$

The following waveforms show the basic pulsed Doppler technique extended to incorporate In-phase and quadrature detection so that the direction of travel can be determined. In this figure it can be seen that the Quadrature signal leads the In-phase signal for a receding target (a) and that it lags the In-phase signal for an approaching target (b).

Note that the in-phase and quadrature traces have been shifted on the display so that they are easier to interpret.



(a)



(b)

Figure 14.12: I and Q Doppler frequencies for a receding (a) and an approaching (b) target

14.5. Doppler Sensors

Many sensors using both ultrasonic and radar technology make use of the Doppler principle to measure target motion effects. Such sensors can either operate using continuous wave (CW) or pulsed sources.

In general CW sources have no range discrimination, while pulsed sources can determine both range and velocity.

14.5.1. Continuous Wave Doppler Ultrasound

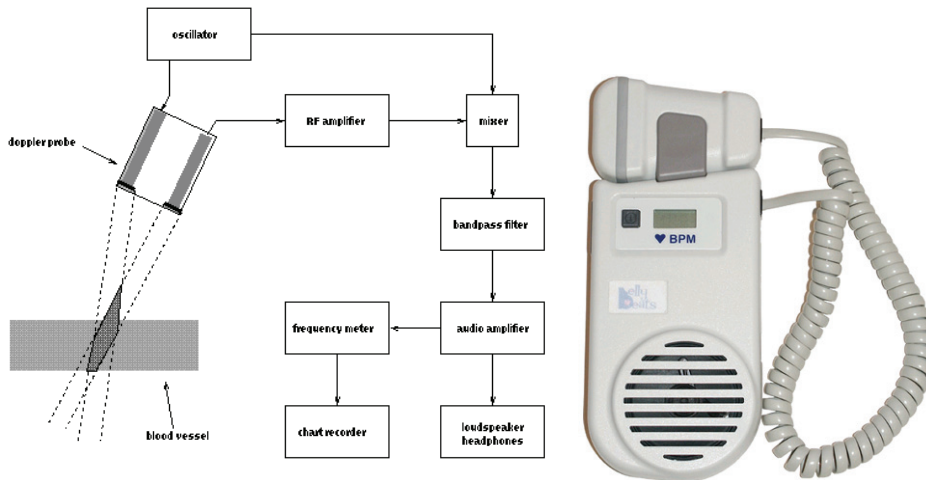


Figure 14.13: Generic CW Doppler instrumentation measuring blood flow in an artery

CW ultrasound systems are used to measure fluid flow and the movement of internal organs (particularly the heart) while industrial Doppler flowmeters measure flow in pipes and canals. In all cases they rely on the acoustic signal reflecting off the moving medium.

If a spectrogram is produced, then it is possible to identify specific target characteristics that vary with time. For example, the following figures show the difference between the return from a large dog and a human subject.

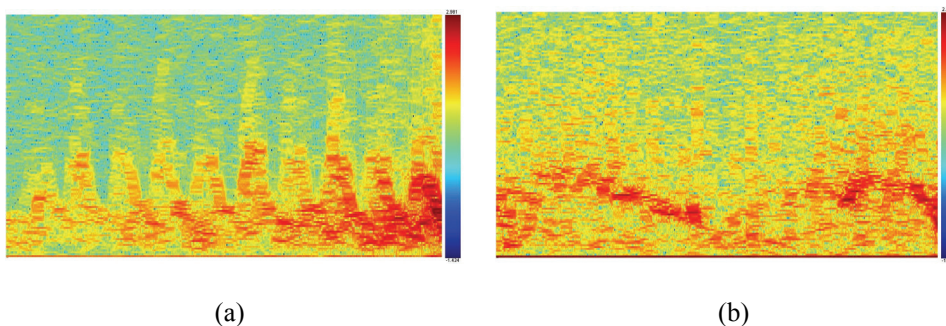


Figure 14.14: Doppler spectrograms of (a) human subject and (b) large dog both walking towards the radar

14.5.2. Continuous Wave Doppler Radar

CW Doppler radar has a myriad of uses. These include short range intruder detection, moderate range sports and police radar for measuring the velocity of moving targets, and long range aircraft tracking applications.

Intruder Detection

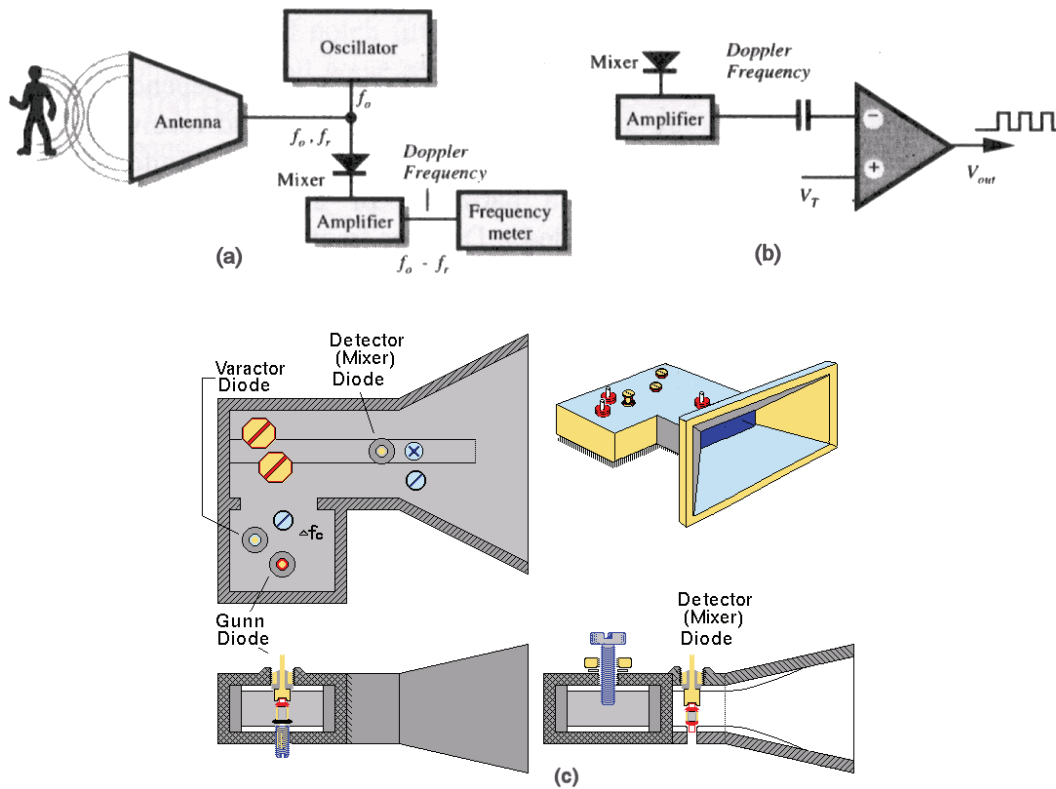


Figure 14.15: CW Doppler intruder detection. (a) a circuit for measuring Doppler frequency, (b) circuit with a threshold detector and (c) Gunn microwave Doppler module

Specifications of a typical intruder system:

- Operational frequency typically X-band (8-12GHz)
- Output power 1-10mW
- Operational principle, Gunn oscillator iris-coupled to the common antenna port that includes a mixer diode.
- Sometimes a varactor diode is mounted in the Gunn diode cavity to allow for electronic control of the oscillation frequency
- Antenna port is generally flared into a horn to constrain the beam angle

Sports Radar

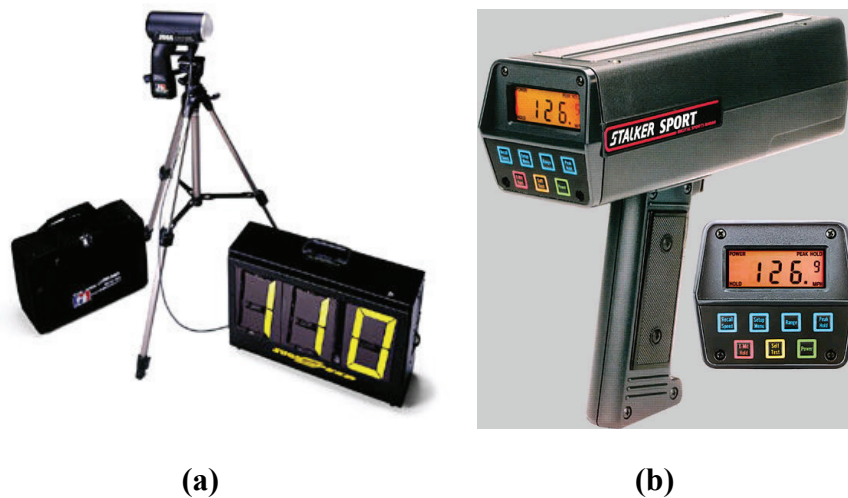


Figure 14.16: Doppler sports radar units, (a) tripod mounted and (b) hand held

Specifications of a typical sports radar

- Operational frequency typically K-band (24.15GHz)
- Output power 40-100mW
- Antenna generally a large horn or a horn-lens
- Range depends on antenna size, output power and target RCS
- Accuracy +/-1km/h (typical)

Police Speed Trap



Figure 14.17: Police radar

Specifications of a typical police radar

- Operational frequency, X-band, K-band or Ka-band
- Accuracy +/- 0.2% Microwave +/- .15 MPH internal signal processing
- +1/4, -3/4 mph readout truncation system accuracy at 100 MPH +0.6/-1.1 mph
- Patrol speed range +16 to +75 mph
- Target Speed Range Stationary/Moving 15 mph thru 199 mph
- Target detection distance: Approx. 1000m.

Aircraft Tracking Radar

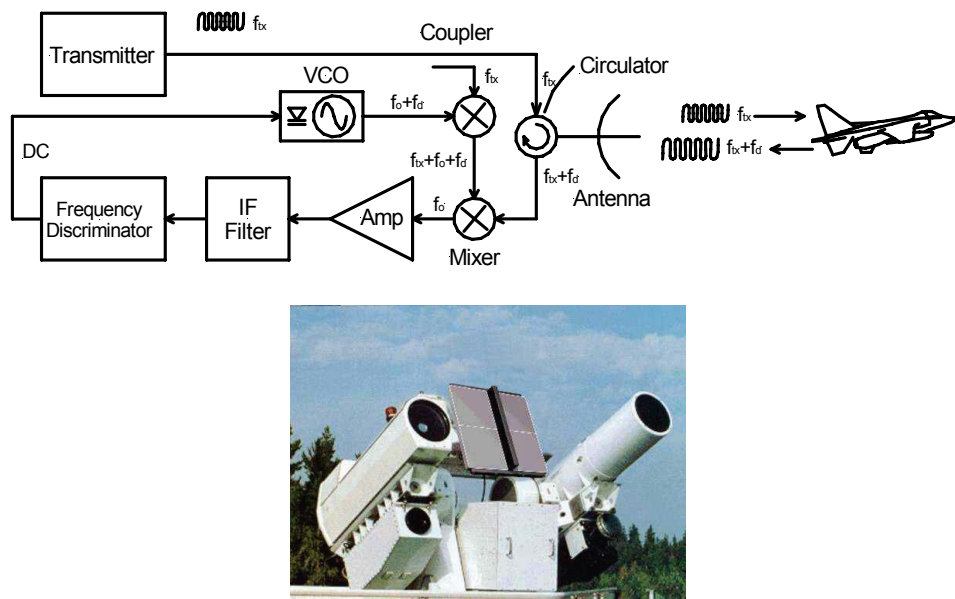


Figure 14.18: CW Doppler aircraft tracking

The block diagram shows a simple aircraft tracking Doppler radar similar to the radar shown in the photograph. The antenna is tracked manually in angles using one of the telescopes. Automatic tracking is carried out only on the velocity. A narrow-band tracking filter in the IF is used to reject noise and crosstalk from the transmitter. This filter is followed by a discriminator that provides an error signal that drives an automatic frequency control (AFC) loop which ensures that the received frequency ($f_{tx} + f_d$) is downconverted to a constant IF, f_o , irrespective of the actual Doppler frequency.

There will be a minimum Doppler frequency below which the transmitter spillover will swamp the received signal.

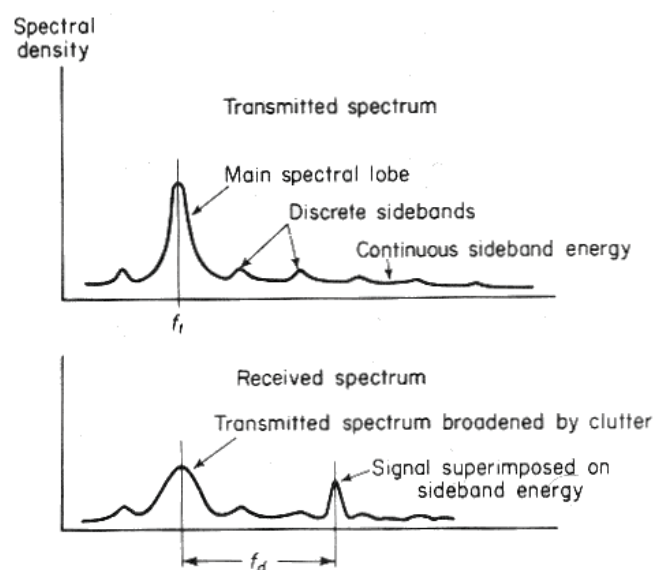


Figure 14.19: CW Doppler spectra for the tracking radar example

14.5.3. Pulsed Doppler Ultrasound



Figure 14.20: Doppler ultrasound of blood flow through the heart

Conventional CW sensors are unable to measure range, and so they are unable to separate Doppler signals returned from different targets within the beam except when beam pairs are used as shown earlier in this chapter.

The primary differences between CW Doppler and pulsed Doppler are as follows:

- A single transducer can be used for transmission and reception because the two are separated in time
- Pulsed Doppler can be incorporated into conventional pulsed echo ultrasound (often known as duplex scanning)
- Periodic bursts of ultrasound typically only a few cycles long are used.
- Pulsed Doppler is only sensitive to flow within a region termed the sampling volume.

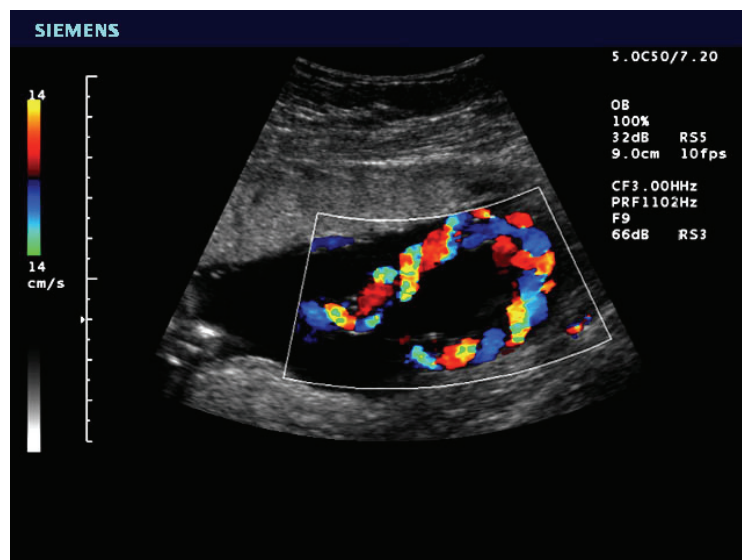


Figure 14.21: Doppler ultrasound image of umbilical cord

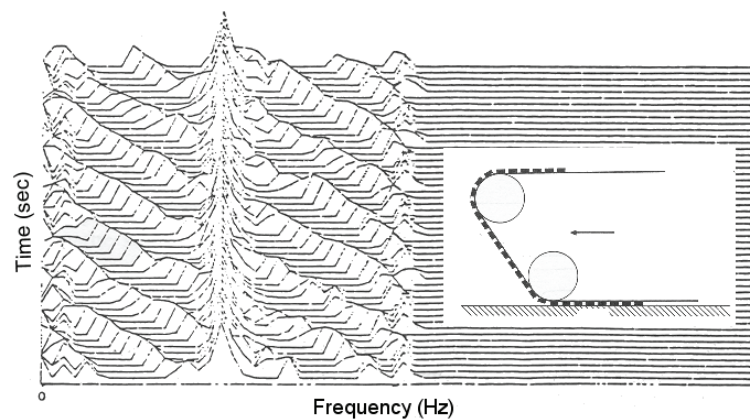
Range resolution is achieved by transmitting a short burst of ultrasound.

Following the burst, the received signal is mixed with a delayed version of the transmitted pulse as reference. The time of flight of the transmitted pulse to the target of interest and back again is equal to this delay. This allows the sampling volume to be moved to different positions along the beam by altering the delay, and in this way, flow at different depths can be selectively monitored. The width of the sampling volume is equal to the transmitted beamwidth, and the length is equal to the pulse width.

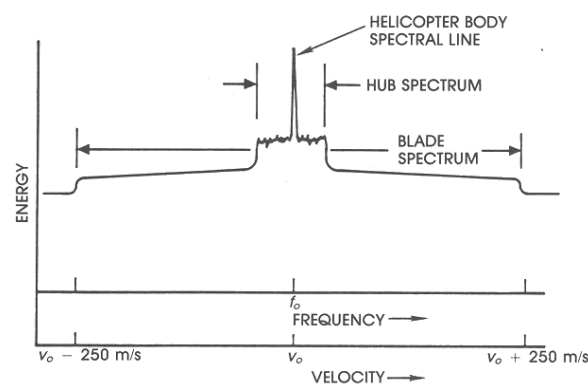
Imaging systems such as the one shown in the figure process the Doppler in a number of consecutive range bins by running a reference CW oscillator continuously as shown in the diagram.

14.5.4. Doppler Target Identification

Doppler signatures of moving targets can be used to differentiate between target types at long range. From a military perspective, this discriminatory capability is extremely useful, for example a high cost missile seeker needs to know whether the target it has detected is a high-value tank or helicopter and a low-value truck.



(a)



(b)

Figure 14.22: Idealised Doppler spectra of (a) moving tank and (b) helicopter that can be used to discriminate between different classes of moving targets

14.5.5. Pulsed Doppler Radar



- Pulsed Doppler radar systems such as the NEXRAD weather surveillance radar shown here operate using similar principles to the imaging Doppler ultrasound discussed above.
- The RF return signal in each range gate is down-converted to baseband using the I/Q principle described earlier in this section.

Figure 14.23: Doppler weather radar

The amplitude and phase of this echo pulse-train will vary with time as shown in (c) for $f_d < 1/\tau$ in Figure.14.11. The Doppler spectrum is extracted from the time waveform of each gate by processing blocks of data through a complex FFT algorithm. The frequency and phase information that is output from each range gate is converted to a radial velocity component from which actual wind velocity can be inferred from the variation in radial velocity of adjacent gates and an understanding of airflow dynamics.

Range and velocity, or range and reflectivity maps such as the one displayed below can then be produced by displaying the data for all range gates over the 360° scan angle of the radar.

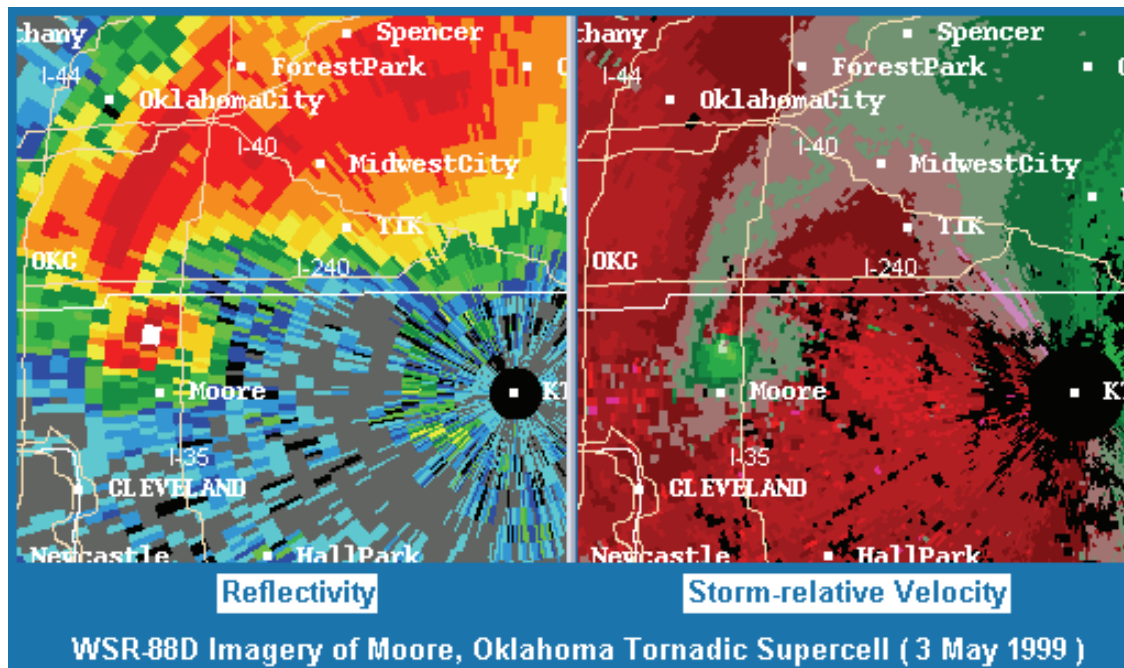


Figure 14.24: Pulsed Doppler radar images of a storm over Oklahoma

An integral and important part of RNDSUP (Radar Network and Doppler Services Upgrade) of the Australian Bureau of Meteorology is the implementation of a new

national radar training program for Australian forecasters. As the Doppler radars are implemented in each state, forecasters will undergo extensive radar training.

The image below is taken from the Doppler radar at Buckland Park. The air is moving in the directions shown by the arrows.

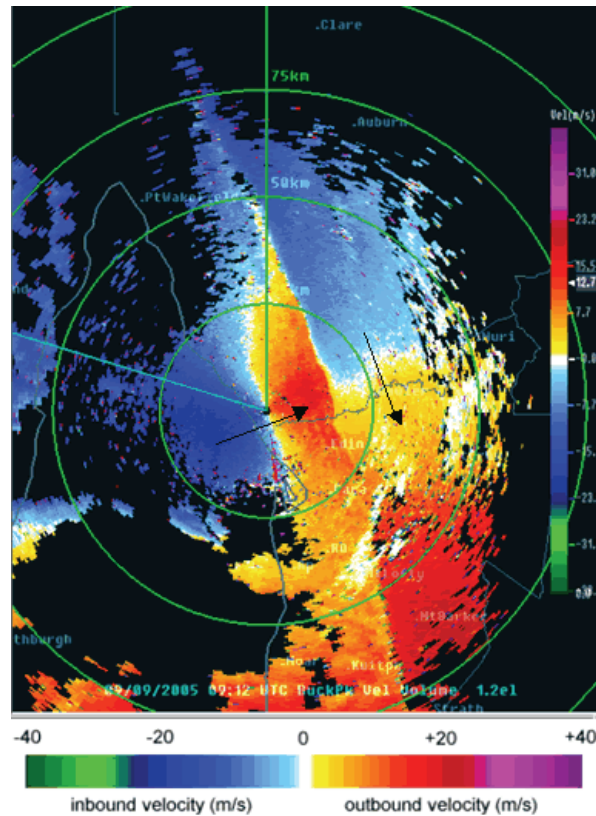


Figure 14.25: Doppler radar display from the Australian bureau of meteorology used to improve weather forecasting

14.6. References

- [1] D.Liley, The Doppler Principle, <http://liley.physics.swin.oz.au>, 03/03/2001.
- [2] Sports radar Gun, <http://www.radarguns.com/specs/radar2000.html>, 22/02/2000.
- [3] Weather Surveillance Radar, <http://www.mws.noaa.gov/modernize/88dtech.html>, 23/02/2000.
- [4] D.Barton, Radar Systems Analysis, Artech, 1976
- [5] How Stuff Works: Doppler Ultrasound, <http://howstuffworks.com>, 4/4/2001.

