Chapter 15.

Tracking Moving Targets

15.1. Track While Scan (TWS)

The track of a target in 2D can be determined from a surveillance radar Plan Position Indicator (PPI) display by plotting the target coordinates as they move when measured from scan to scan.

At its most simple, this tracking function can be performed by a radar operator marking the face of the cathode ray tube with a pen. This is an inaccurate process and limits the number of targets that can be handled at one time.

Figure 15.1: Examples of plan position indicator display hardware

Figure 15.2: Plan position indicator showing a simulated and a real display
Automatic trackers operate as follows (for a single target)

1. Target is detected as the received echo exceeds a threshold. There is no information about its velocity. The software constrains the uncertainty to a reasonable value for an aircraft target (large circle).

2. Target is detected again displaced in range and angles, but within the uncertainty boundary. A crude velocity estimate is made and the position where the target will appear next is predicted. The uncertainty is still large as the position and velocity estimates are not good.

3. The target appears within this uncertainty boundary. Tracking filters estimates of position and velocity improve, and the next sample prediction is made with a smaller position uncertainty.

4. As with (3)

5. The actual target position falls outside the position uncertainty boundary because it has accelerated, and the prediction algorithm only used position and velocity. Track is lost.

6. A new target is detected with unknown velocity.

Figure 15.3: PPI Display sequence to illustrate the target tracking process

One method of performing the filtering and prediction function is to use $\alpha\beta$ filters (see Chapter 13) for the polar co-ordinates $(R, \theta)$. An alternative would be to convert the measured positions from polar to Cartesian $(x, y)$ before filtering as shown in the figure.

Figure 15.4: Track while scan processing
15.2. The Coherent Pulsed Tracking Radar

Pulsed tracking radar systems, such as the BAE Systems unit shown in the figure below, are mostly used for military applications such as fire control. These radars track fast moving aircraft or missiles with high accuracy, and then use the estimates of the target position and velocity to direct missiles or anti-aircraft guns.

Such coherent radar systems extract both amplitude and phase information from the signal reflected by a target. This is required because the length of a single pulse is too short to resolve typical target Doppler frequencies as shown in (c) of the figure below.

To extract Doppler shift, the returns from many pulses over an observation time $T$ must be analysed so that the spectrum can be resolved down to a bandwidth $\beta \approx 1/T$. For this process to work, a deterministic phase relationship must be maintained over the observation time $T$. 
15.2.1. Single Channel Detection

The block diagram of a single channel coherent radar is shown in the figure below.

The transmitter configuration is Master Oscillator Power Amplifier (MOPA)
- The transmitted signal is obtained by mixing an RF signal from a Stable Local Oscillator (STALO) with a frequency $f_{RF}$ with a coherent oscillator (also very stable) with a frequency $f_{COH}$.
- The resulting frequency (after filtering) id $f_o$ is pulse modulated to form a pulse train, amplified and transmitted.

The received signal is down converted to IF (typically 30-60MHz) by mixing with the STALO signal. After amplification and filtering it is down converted further to baseband (video) by mixing with the COHO signal.

A consequence of this single channel down conversion is that there is no direction information in any Doppler modulation since a target receding would produce exactly the same Doppler signature as one approaching and the response is

$$V_i = k \sin(2\pi f_d t + \phi_o),$$

where $V_i$ – Video output voltage,
$k$ – Amplitude of the video signal,
$f_d$ – Doppler frequency (Hz),
$\phi_o$ – Phase shift (rad).

Figure 15.7: Single channel coherent pulsed Doppler radar
15.2.2. I/Q Detection

In an I/Q detector, the IF signal is split into two channels with the quadrature (Q) channel being phase shifted by 90° with respect to the in-phase (I) channel.

Though the two Doppler signals output by the I and Q channels will have identical frequency whether the target is approaching or receding, their phase relationship with each other will reverse, and so direction information can be obtained as discussed in Chapter 13.

![Figure 15.8: Coherent pulsed radar with a synchronous (i/q) detector](image)

By sampling the I and Q outputs and using a complex fast Fourier Transform (FFT), the magnitude and phase of the combined Doppler spectrum can be obtained.

Another benefit of I/Q detection over single channel detection is that a 3dB gain in SNR is obtained at the output of the Doppler spectral analysis function.

If the actual Doppler frequency is not important, but only the fact that the target is moving at important, then a process called Moving Target Indication (MTI) can be applied.

15.2.3. Moving Target Indicator (MTI)

For a moving target, if the video output of the I or Q channel is examined, the amplitude will vary on a pulse to pulse basis, as shown earlier, due to the changing phase between the transmitted and received signals. For a static target, the phase will remain unchanged, and the amplitude will remain constant.

An MTI based on a delay-line canceller operates by taking the difference of the amplitudes of successive pulses as shown below.
This is in effect a Finite Impulse Response (FIR) filter with a high-pass characteristic that rejects signals that are unchanging or that are changing very slowly. However, because it becomes in effect a sampled data system, the frequency response is repeated with a period of $1/T$ as shown in the figure below.

Standard FIR filter techniques can be used to adjust the weightings of a canonical filter implementation, as shown below, to alter the filter response and narrow the stop band and flatten the pass band.

One of the disadvantages of classical MTI techniques is that they cannot discriminate between a moving target and moving clutter, nor do they cope very well if the radar is also moving.
Blind Speeds

If the target Doppler frequency lies in the region where \( f_d \approx \frac{1}{T} \), it can be seen, from the transfer function above, that it is attenuated. Zeros also occur at \( \frac{2}{T} \), \( \frac{3}{T} \), \ldots \( \frac{n}{T} \)

\[
f_d = \frac{n}{T} = nf_p,
\]  \hspace{1cm} \text{(18.2)}

where \( f_d \) – Doppler frequency (Hz),
\( f_p \) – Pulse repetition frequency (PRF) (Hz).

The actual blind speeds are given by the following

\[
v_n = \frac{n\lambda}{2T} = \frac{n\lambda f_p}{2} \text{ m/s},
\]  \hspace{1cm} \text{(18.3)}

where \( v_n \) – Blind speed (m/s),
\( \lambda \) - Transmitter wavelength (m),
\( n = 1,2,3,\ldots \)

To obtain a high first blind speed, it is necessary to either operate with a long wavelength (often not practical), or to operate with a high PRF. However, a high PRF becomes ambiguous at a short range which is also not ideal for surveillance or long range tracking radar applications.

Staggered PRF and Blind Speed

The effect of blind speeds can be reduced by operating at more than one PRF as shown below.

Figure 15.12: Effect of staggered PRF on MTI transfer function
As the ratio of the pulse repetition interval, PRI, \( T_1/T_2 \) approaches unity, the greater will be the value of the first blind speed. However, the first null also gets deeper and so the rejection of slowly moving clutter will be compromised.

To cater for the moving clutter problem, a bank of Doppler filters can be implemented instead of a delay line canceller. In modern radars, this process is generally implemented digitally using the Complex FFT. The outputs (excluding sidelobes) of such a filter implementation are shown below.

To obtain sufficient rejection of unwanted signals in adjacent bins, the filter sidelobes must be made as low as possible.

![Figure 15.13: Filter bank implemented using the FFT](image)

A tradeoff exists between the width of the mainlobe and the sidelobe level with different windowing (weighting) functions as discussed in Chapter 11.

<table>
<thead>
<tr>
<th>Weighting Function</th>
<th>Peak Sidelobe Level (dB)</th>
<th>MainLobe Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>-13.26</td>
<td>0.886</td>
</tr>
<tr>
<td>Hanning</td>
<td>-31.5</td>
<td>1.42</td>
</tr>
<tr>
<td>Hamming</td>
<td>-42.5</td>
<td>1.32</td>
</tr>
<tr>
<td>Taylor n=5</td>
<td>-34</td>
<td>1.19</td>
</tr>
<tr>
<td>Taylor n=6</td>
<td>-40</td>
<td>1.25</td>
</tr>
<tr>
<td>Dolph Chebyshev</td>
<td>-40</td>
<td>1.2</td>
</tr>
</tbody>
</table>

15.3. Limitations to MTI Performance

Sub-clutter visibility is the ratio by which the moving-target power may be lower than the clutter in the same range bin, and still be detected with a specified \( P_d \) and \( P_{fa} \).

It is usually limited by internal instabilities in the amplitude and phase of the various waveforms generated by the radar, by the motion of a scanning antenna and the finite time on target which has the effect of widening the clutter spectrum and also by the bandwidth characteristics of real clutter. Rain is blown by the wind, the sea moves, as do leaves and grass.

15.4. Range Gated Pulsed Doppler Tracking

The Doppler tracking loop and the range tracking loop are interrelated. The Doppler tracking loop operates on video sampled at the range indicated by the range tracker while range tracking is usually accomplished by means of the split (early/late) gate technique.
Before the signals in the early and late gates are compared to derive the range tracking error, each is passed through a Doppler tracking filter to reject returns from stationary objects at the same range.

The range tracking loop can be updated at the PRF unless an FFT that processes $N_p$ pulses is used to perform the Doppler analysis in which case the update frequency is reduced to $\text{PRF}/N_p$.

![Schematic diagram of a range gated doppler tracker](image)

**Figure 15.14: Schematic diagram of a range gated doppler tracker**

Initiation of tracking requires the near simultaneous initialisation of all four tracking coordinates: range, two angles and Doppler frequency, this is achieved as follows.

- The operator who identifies a target during the search phase designates the selected target on the display using a cursor.
- On the following scan, if the target is still present, the range, azimuth and elevation angles and Doppler frequency are recorded.
- The scan sequence is interrupted and the radar antenna returns automatically to the designated area.
- The range gate pair are moved to the designated range where they begin a small search to compensate for uncertainties in the designation.
- The sum channel video received by the combined early and late gates, or by a third target gate that straddles the two is processed by a Doppler analyser (eg FFT)
- If a moving target is found at the designated range and sufficiently close to the Doppler of the detection that triggered the acquisition sequence, then the Doppler tracking gate is placed over the target.
- The Doppler loop is closed by using a frequency discriminator to maintain the target in the centre of the tracking filter.
- Video signals in both the early and late gates are analysed by Doppler filters slaved to the frequency of the Doppler tracker.
- After allowing the Doppler loop time to settle, the range tracking loop is closed by using the difference in the relative amplitudes of the early and late gates to control the range position.
• The video signals in both the azimuth and elevation monopulse angle error channels are sampled by a range gate slaved to the target gate of the range tracker.
• The sampled video for each of the two error signals is filtered by a Doppler filter slaved to the frequency of the Doppler tracker. Thus angle errors are derived only from the same range-Doppler cell that contains the target.
• After a brief period to allow the range tracking loop to settle, the angle tracking loop is closed.

The transition from search to track is not a trivial problem, and it is made even more difficult in the military scenario, if the target either starts to manoeuvre or tries to disrupt the process by deploying chaff or by some electronic means.

15.5. Co-ordinate Frames

15.5.1. Measurement Frame

Radar measurements are made in polar space \( (R,\theta,\phi) \) as the radar can only measure range, elevation and bearing (azimuth).

15.5.2. Tracking and Estimation Frame

The equations of motion that govern the profile of a target operate in Cartesian space, \( (x,y,z) \), so it is advantageous to transform the co-ordinate system from polar to Cartesian space.

A point \( P \) can be located by spherical coordinates \( (r,\theta,\phi) \) as well as rectangular coordinates \( (x,y,z) \).

The transformation between those coordinates is

\[
\begin{align*}
x &= r \sin \theta \cos \phi \\
y &= r \sin \theta \sin \phi \\
z &= r \cos \theta \\
r &= \sqrt{x^2 + y^2 + z^2} \\
\phi &= \tan^{-1}(y/x) \\
\theta &= \cos^{-1}(z/\sqrt{x^2 + y^2 + z^2})
\end{align*}
\]

![Figure 15.15: Rectangular <-> Polar (spherical) transfer function](image)

Generally this frame of reference will remain centred at the radar, however, some fire control systems translate and rotate the frame to make it target centred as the aircraft dynamics can be better modelled in this frame.

If more than one sensor may be involved in the tracking function, and these sensors are not co-located (they may be on different platforms that move relative to each other), then an earth centred Cartesian frame is generally used.
The pencil beam of a tracking radar must be pointed at the target for tracking to occur. This is quite a challenge as a typical tracking radar has a 3dB beamwidth between 1° and 2°, or smaller in the case of the radar for a close-in weapon system onboard ship.

A servo system is used to drive the antenna in the direction that minimises the tracking errors. Most servo systems are Type II, or zero velocity error systems since, in theory, no steady-state error exists for a constant velocity (angular rate) input. With Type II systems, dynamic lags proportional to the magnitude of the target acceleration do occur. To accommodate this, the tracking bandwidth is adjusted to minimise the tracking error which is due to a combination of measurement noise and dynamic lag.
At long range where the angular motion of the target is small, a very small tracking bandwidth can be tolerated. However, at short range where target angular rates and accelerations are large, a wider bandwidth becomes acceptable. Secant correction increases the azimuth error signal gain as a function of the elevation angle.

Another restriction on tracking bandwidth is that it must be small (10%) compared to the lowest natural resonant frequency of the antenna and mounting structure to reduce the risk of instabilities occurring.

Figure 15.17: Monopulse tracking radar

Figure 15.18: Lowest resonant frequency as a function of antenna diameter
The pointing system, generally known as a pedestal must be able to accommodate both target motion as discussed and its own base motion (if mounted on a moving vehicle).

There are many different types of antenna mounts that can be used depending on the tracking and stabilisation requirements.

15.6.1. On-Axis Tracking

The best tracking occurs using null-steering when the antenna is pointed towards the target with an accuracy of only a few milliradians. This is known as on-axis tracking.

It reduces cross coupling between the axes by minimising cross polar levels and reducing the effects of system nonlinearities.

It requires the following:
- the removal by prior calibration of biases
- a filter than can perform one sample ahead prediction
- the selection of the appropriate co-ordinate system for tracking.

Target dynamics that dictate the real and apparent acceleration and tracking loop bandwidth determines the tracking accuracy.

15.6.2. Crossing Targets and Apparent Acceleration

![Diagram of crossing target](image)

Figure 15.19: Geometry of a crossing target (a) in plan and (b) in perspective
When viewed in radar polar co-ordinates, the target velocities will be

$$\dot{R}_{\text{max}} = v_i, \quad \dot{A}_{\text{max}} = \frac{v_i}{R}, \quad \dot{E}_{\text{max}} = \frac{v_i}{R}.$$  \hspace{1cm} (18.4)

The real accelerations are

$$\ddot{R}_{\text{max}} = a_i, \quad \ddot{A}_{\text{max}} = \frac{a_i}{R}, \quad \ddot{E}_{\text{max}} = \frac{a_i}{R}.$$  \hspace{1cm} (18.5)

The geometric accelerations and other derivatives are shown below. They are normalised in the angular co-ordinates to

$$A_{\text{max}} = \sigma_m = v_i / R_c.$$ Where $R_c$ is the ground range at crossover.

![Derivatives of azimuth for pass course.](image1)

![Derivatives of elevation angle for pass course ($X = 1$).](image2)

Figure 15.20: Angular derivatives for crossing targets

In range, the derivatives are normalised to $R_a = R_{\text{min}}$, at the point of closest approach.
Conventional servomechanism theory can be used to determine the lag errors in radar tracking loops if conventional loops are implemented.

The lag error can be written as follows

$$\delta_a = \frac{\sigma_a}{K_v} + \frac{\dot{\sigma}_a}{K_a} + \frac{\ddot{\sigma}_a}{K_3} + \ldots$$  \hspace{1cm} (18.6)

Where the coefficients $K_v$, $K_a$ and $K_3$ are the servo error coefficients, the values of which increase with increasing loop gain and bandwidth.

Servos are classified according to the first coefficient that is finite in the loop design. A type 1 servo has a finite $K_v$ (but infinite position error constant $K_o$), a type 2 has finite $K_a$ but infinite $K_v$, etc.

The acceleration error coefficient is intimately connected to the closed loop bandwidth of the servo. If the bandwidth is expressed in terms of the equivalent noise bandwidth $B_n$ then

$$K_a = 2.5B_n^2 = 0.63/t_o^2.$$  \hspace{1cm} (18.7)

Where $t_o$ is the equivalent averaging time for the tracking loop. The lag will be

$$\varepsilon_a = \frac{\ddot{\sigma}_a}{K_a} = \frac{\ddot{\sigma}_a}{2.5B_n^2} = 1.6\sigma_a^2t_o^2.$$  \hspace{1cm} (18.8)

This formula is applicable to azimuth, elevation and tracking lags with their corresponding acceleration components.
If thermal noise and dynamic lag are the primary sources of error, then an optimum bandwidth can be obtained to minimise the total error variance. According to Barton, this is

\[
\sigma_\theta^2 = \frac{\theta_i^2 B_n}{k_m^2 f_r B \tau S / N} + \frac{a_t^2}{6.3 R^2 B_n^2},
\]

(18.9)

\[
B_n = \left[ \frac{a_t^2 k_m^2 f_r B \tau S / N}{1.6\theta_\theta R^2} \right]^{1/5},
\]

(18.10)

where:
- \( a_t \) – acceleration (real or geometric) (m/s²),
- \( k_m \) – Monopulse Gain Constant (typ 1.6),
- \( f_r \) – Pulse repetition frequency (Hz),
- \( B \tau \) - IF bandwidth and pulse width (see matched filter),
- \( S / N \) – Single pulse signal to noise ratio,
- \( \theta_\theta \) – Antenna beamwidth (rad),
- \( R \) – Crossing Range (m).

\[\text{RMS Angle Tracking Error}\]

\[\text{RMS Tracking Error (mrad)}\]

\[\text{Bandwidth (Hz)}\]

\[\text{Thermal Noise, Dynamic Lag, Combined}\]

**Figure 15.22: Typical noise and lag optimisation**

### 15.6.3. Tracking in Cartesian Space

One method of maintaining the noise performance of the system while minimising the dynamic lag is to operate with a wide angle-servo bandwidth, and to perform the tracking and smoothing in Cartesian space.

As there are no geometric accelerations in Cartesian space, it is possible to reduce the filter bandwidth to less than 1Hz. This determines the noise performance of the tracker.
Because the angle servo bandwidth is wide (typically 10 to 100Hz for a real system), the dynamic lag for geometric accelerations is limited.

A simplified block diagram showing the loop configuration is shown below.

Figure 15.23: Tracking in Cartesian space
15.7. Fire Control Radar Design

15.7.1. Requirements

Ships Motion
- Radar on the deck of a ship 8m above the water
- Roll +/-25° Pitch +/- 10° Yaw +/-5°
- Period about 5s

Designation
- From a surveillance radar (one time per second)
- Elevation accuracy +/-5°, Azimuth accuracy +/-2°
- Range accuracy +/-25m
- No velocity information

Environment
- Up to sea state 5 (very rough, wave height 2.4 to 3.6m)
- Rainfall up to 25mm/hr

Target Types
- Fixed Wing Aircraft (frontal RCS 1m² independent of frequency)
- Aircraft Height > 60m
- Sea Skimming Missiles (frontal RCS 0.1m² independent of frequency)
- Sea Skimmer height 3m

Detection Performance
- Probability of detection $P_d = 0.95$
- Probability of false alarm $P_{fa} = 10^{-6}$
- Detection time from receipt of designation (excluding slew) 0.5s
- Detection Range

<table>
<thead>
<tr>
<th>Weather</th>
<th>Aircraft</th>
<th>Sea Skimmer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear Air</td>
<td>15km</td>
<td>6km</td>
</tr>
<tr>
<td>Rain 12.5mm/hr</td>
<td>10km</td>
<td>4km</td>
</tr>
<tr>
<td>Rain 25mm/hr</td>
<td>5km</td>
<td>2km</td>
</tr>
</tbody>
</table>

Tracking Performance
- Minimum tracking range $R_{min} = 50m$
- Range tracking accuracy (<1m RMS)
- Angle tracking accuracy (<1mrad RMS)
- Tracking aircraft directly overhead at $h>100m$
- Aircraft and missile velocity <280m/s

Safety Constraints
- Average Transmit Power <100W
15.7.2. Selection of Polarisation

For low flying aircraft over the sea we want to minimise the sea clutter. In calm seas at low grazing angles the difference between the reflectivity for horizontal polarisation and vertical polarisation exceeds 12dB. As the roughness increases the difference decreases.

![Composite of σ° data for a "medium" sea.](image)

**Figure 15.24: Sea Clutter reflectivity as a function of grazing angle**

The radar will operate using horizontal polarisation, and it will be assumed that the surface reflectivity $\sigma^o$ is $-35$dB.

15.7.3. Pedestal Specifications

The maximum combined roll and pitch angle is

$$\varphi_{\max} < \sqrt{25^2 + 5^2} .$$

This does not exceed $30^\circ$
The antenna can be stabilised with respect to this tilt adequately without resorting to a 3rd axis, however, it can result in a significant rotation of the polarisation.

The pedestal will be of the type elevation, $\theta$, over azimuth, $\phi$.

$\theta$ defined as +ve up from the horizontal

$\phi$ defined as +ve anticlockwise from the x-axis

For a mounting height $\approx 8\text{m}$ above the sea with the minimum tracking angle for a sea skimmer at $R = 50\text{m}$ ($\theta = -6^\circ$) and a combined roll and pitch angle of $30^\circ$ requires a minimum angle of $-36^\circ$ ($\theta_{\text{min}} = -40^\circ$).

To allow the antenna to track over the vertical, and to have time to slew around in azimuth without losing lock at the maximum combined roll and pitch angle requires $\phi_{\text{max}} = 90 + 30 = 120^\circ$ (use $125^\circ$)

15.7.4. Radar Horizon

For $h_r = 8\text{m}$ and $h_t = 60\text{m}$, the radar horizon is given by

$$d = 130(h_r(\text{km}) + h_t(\text{km}) = 43\text{km}$$

The radar horizon is not a consideration for this design.

15.7.5. Selection of Frequency

A reasonable maximum diameter for the antenna on a shipboard radar is 1.5m.

The beamwidth at various frequencies will be as follows:

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Band</th>
<th>Beamwidth (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>X</td>
<td>1.4</td>
</tr>
<tr>
<td>35</td>
<td>Ka</td>
<td>0.4</td>
</tr>
<tr>
<td>94</td>
<td>W</td>
<td>0.13</td>
</tr>
</tbody>
</table>

A narrow beam decreases the effect of multipath and limits the area of clutter within the tracking gate, while attenuation increases with frequency (particularly in the rain)

The minimum elevation angle when tracking at target at $h = 60\text{m}$ and $R = 15\text{km}$ $\theta = 0.19^\circ$ and the minimum tracking angle when tracking a sea skimmer at $h = 3\text{m}$ and $R = 50\text{m}$ $\theta = -6^\circ$

Though it may be possible to minimise the effects of multipath, it is not possible to eliminate them when the radar is looking down at the target.
Multipath has a major effect on tracking accuracy as shown by the following measured data for two systems tracking the same target.

The best compromise is to use the narrowest beamwidth possible and to use multipath reduction techniques. According to the literature application of these techniques can maintain an RMS elevation tracking accuracy of between 0.05 and 0.1 beamwidths.

Assuming that we can only manage 0.1 beamwidths, then the tracking accuracy will be:

<table>
<thead>
<tr>
<th>Band</th>
<th>Beamwidth (deg)</th>
<th>RMS Tracking (mrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Ka</td>
<td>0.4</td>
<td>0.69</td>
</tr>
<tr>
<td>W</td>
<td>0.13</td>
<td>0.22</td>
</tr>
</tbody>
</table>

This excludes the X-band option as the tracking accuracy does not meet the accuracy criteria defined earlier.
15.7.6. Adverse Weather Effects

Typical attenuation as a function of frequency under different weather conditions is shown below:

Table 15.5: Atmospheric attenuation

<table>
<thead>
<tr>
<th>Band</th>
<th>Clear (dB/km)</th>
<th>Rain 12.5mm/h (dB/km)</th>
<th>Rain 25mm/h (dB/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.02</td>
<td>0.25</td>
<td>1</td>
</tr>
<tr>
<td>Ka</td>
<td>0.15</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>W</td>
<td>0.3</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

15.7.7. Required Single Pulse Signal to Noise Ratio

The required S/N to achieve the specified $P_d$ and $P_{fa}$ is a function of the target distribution and its fluctuation characteristics. For a non-fluctuating target this is 13.6dB.

Figure 15.27: Detection probability curves
Swerling 1: Many independent scatters of similar RCS. This results in slow fluctuations with time.

Swerling 2: One major scatterer and many smaller scatterers. This is typical of an aircraft nose on. This results in fast fluctuations with time.

Figure 15.28: Fluctuating target effects

The additional SNR required to achieve these probabilities of the target is fluctuating is determined from this graph. It will be 10.4dB. The single pulse SNR required is thus 13.6+10.4 = 24.0dB.

15.7.8. Tracking Gate Size

Ideally, the tracking gate size is matched to the target length

A typical Jet fighter would be 15m long, so a gate size between 15 and 20m would be ideal. We will use 20m.

This minimises the amount of clutter received, it does however complicate the target acquisition process as the designation accuracy is only +/-25m, so at least 3 gates would be required to span the uncertainty

15.7.9. Signal to Clutter

We assume that the reflectivity of the sea is the same at X, Ka and W bands

We assume that a similar SCR is required as SNR

For a gate size of 20m, the illuminated area will be a function of the antenna beamwidth
For an average reflectivity $\sigma^o = -35$dB in all cases, the clutter RCS will be

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Band & RCS (6km) \text{(dBm}^2\text{)} & RCS (15km) \text{(dBm}^2\text{)} \\
\hline
X & -0.33 & 3.6 \\
Ka & -5.8 & -1.8 \\
W & -10.8 & -6.8 \\
\hline
\end{tabular}
\caption{Clutter RCS as a function of frequency}
\end{table}

For an aircraft RCS of $1\text{m}^2$ (0dBm$^2$) or a sea skimmer with an RCS of $0.1\text{m}^2$ (-10dBm$^2$), we are looking for an extra 20 to 30dB of signal to achieve the required SCR of 24dB (for the required single pulse detection probability).

Integration improvement cannot be used for improving the SCR because, unlike the white thermal noise used for target detection, clutter is correlated, so integration will not be as effective.

The primary difference between the targets and the clutter is that generally the target has a significant radial velocity and the clutter is static (or slow moving in high seas).

\subsection*{15.7.10. Moving Target Indicator}

Some form of MTI will have to be implemented that can achieve a sub-clutter visibility as follows

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Band & Clutter RCS \text{At 6km} \text{(dBm}^2\text{)} & Sea skimmer RCS \text{(dBm}^2\text{)} & Subclutter Visibility \\
\hline
X & -0.33 & -10 & 33.67 \\
Ka & -5.8 & -10 & 28.2 \\
W & -10.8 & -10 & 23.2 \\
\hline
\end{tabular}
\caption{Subclutter visibility requirements sea skimmer}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Band & Clutter RCS \text{At 15km} \text{(dBm}^2\text{)} & Aircraft RCS \text{(dBm}^2\text{)} & Subclutter Visibility \\
\hline
X & 3.6 & 0 & 27.6 \\
Ka & -1.8 & 0 & 22.2 \\
W & -6.8 & 0 & 17.2 \\
\hline
\end{tabular}
\caption{Subclutter visibility requirements aircraft}
\end{table}

This can be achieved using a delay-line canceller, however because the ship will be moving, the clutter will also have an effective velocity, so this is not a good technique to use.
A better alternative is to take more samples, to window them and to use an FFT to isolate moving from “static” targets.

With a Hamming window, a static return rejection of 42dB can be obtained. However, it is unlikely that if the measurement is unambiguous in range, that it will be unambiguous in velocity.

15.7.11. The Pulse Repetition Frequency

To be unambiguous in range out to 15km, the maximum allowed PRF is calculated as

$$ PRF = \frac{c}{2R_{\text{max}}} = 10\text{kHz} $$

Using the Nyquist criterion, this is unambiguous in velocity up to a Doppler frequency of 5kHz

The maximum unambiguous radial velocity is given by the following formula:

$$ v_r = \frac{f_d \lambda}{2} = \frac{PRF \cdot \lambda}{4} $$

<table>
<thead>
<tr>
<th>Band</th>
<th>Unambiguous Velocity $v_r$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>75</td>
</tr>
<tr>
<td>Ka</td>
<td>21.5</td>
</tr>
<tr>
<td>W</td>
<td>8</td>
</tr>
</tbody>
</table>

All velocities will be folded down into this range, so it is possible that the folded target velocity will fall directly into the bin (or bins) containing the clutter.

This can be ameliorated by changing the PRF on a block by block basis, which will shift the relative positions of the clutter and the folded target Doppler frequencies until an one is found in which the target is not in the same bin as the clutter.

It is quite likely that the clutter velocity spread will exceed 8m/s, so there may not be an acceptable PRF at W-Band.

Because both the clutter and the ship may be moving, the actual gate in which the former will be found must be tracked.

![Figure 15.29: Separating target and clutter returns in a Doppler filter bank](image)
15.7.12. Search Requirement

Need to search a volume $10^\circ \times 4^\circ \times 50\text{m}$ in less than 0.5s to meet the specification.

Any of the following patterns can be adopted

![Search pattern options](image)

**Figure 15.30: Search pattern options**

Option (a) requires that the pedestal changes direction more often than option (b), and there is more of an overlap between scans with option (c), so adopt option (b)

For a 50% overlap between scans, the total distance that must be travelled to search each area is determined from the number of vertical scans required to cover the $4^\circ$ in azimuth.

<table>
<thead>
<tr>
<th>Band</th>
<th>Vertical Scans (n)</th>
<th>Total Distance (deg)</th>
<th>Scan Speed (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>4/1.4x1.5=4.28 [5]</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Ka</td>
<td>4/0.4x1.5=15 [15]</td>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>W</td>
<td>4/0.13x1.5=46 [46]</td>
<td>460</td>
<td>920</td>
</tr>
</tbody>
</table>

Hits per scan (the number of pulses that illuminate the target as the beam passes over it) is determined from the required scan rate and the antenna beamwidth.

<table>
<thead>
<tr>
<th>Band</th>
<th>Hits Per Scan (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>140</td>
</tr>
<tr>
<td>Ka</td>
<td>13.3</td>
</tr>
<tr>
<td>W</td>
<td>1.4</td>
</tr>
</tbody>
</table>

There are not sufficient hits at either Ka band or W band to generate an FFT with sufficient bins.

Decreasing the overlap to 0%
Table 15.13: Scan distance and hits per scan for 0.5s detection time

<table>
<thead>
<tr>
<th>Band</th>
<th>Vertical Scans (n)</th>
<th>Total Distance (deg)</th>
<th>Scan Speed (deg/s)</th>
<th>Hits per Scan (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3</td>
<td>30</td>
<td>60</td>
<td>233</td>
</tr>
<tr>
<td>Ka</td>
<td>10</td>
<td>100</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>W</td>
<td>31</td>
<td>310</td>
<td>620</td>
<td>2.1</td>
</tr>
</tbody>
</table>

There is the added burden that the target may have moved up to 140m during the search, so a large bank of gates spanning the original 50m uncertainty plus 140m on each side (330m) must be examined.

At least 17 gates will be required to span this range.

It is now feasible to use a 16 point FFT at Ka band or a 128 point FFT at X-band.

15.7.13. Integration Gain

The FFT process is the equivalent of a coherent integrator, which will produce gains of $10\log_{10}(N)$. However, because the target is fluctuating, this effect will vary with the observation time compared to the fluctuation rate and the probability of detection.

We use the following graph (which is not quite correct as it assumes post detection integration).

For a $P_d \approx 0.95$ (we use 0.9) and a Swerling 2 target, the integration gain is about 17dB for the 16 point FFT at Ka band.

For the 128 point FFT at X band, the integration gain is about 24dB.

Though these are higher than the theoretical maxima of 12 and 21dB respectively, the extra is taken care of in the 10.4dB required for detection described earlier.
15.7.14. Matched Filter

We assume that the pulse is rectangular and that the filter is made up of 5 cascaded tuned bandpass sections.

For a pulse width of 20m (133ns), the optimum bandwidth $\beta = 5$MHz.

<table>
<thead>
<tr>
<th>Input Signal</th>
<th>Filter</th>
<th>Optimum $\beta \tau$</th>
<th>Loss in SNR compared to Matched Filter (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Pulse</td>
<td>Single tuned circuit</td>
<td>0.4</td>
<td>0.88</td>
</tr>
<tr>
<td>Rectangular Pulse</td>
<td>Two cascaded tuned circuits</td>
<td>0.613</td>
<td>0.56</td>
</tr>
<tr>
<td>Rectangular Pulse</td>
<td>Five cascaded tuned circuits</td>
<td>0.672</td>
<td>0.5</td>
</tr>
</tbody>
</table>

15.7.15. Transmitter Power

The average power allowed is 100W, the pulse width is 133ns and the PRF is 10kHz making the duty cycle 0.133% and so the peak transmitted power $P_t = 75$kW.

This can be achieved using a Magnetron or Travelling Wave Tube at Ka or X band.

15.7.16. System Configuration

A Magnetron can be used in a (pseudo) coherent radar if the LO is primed by the random start phase of the magnetron on every pulse.

![Figure 15.32: Schematic diagram of a pseudo coherent pulsed radar](image)
15.7.17. Free Space Detection Range

Applying the radar range equation when there is significant attenuation is best achieved using MATLAB

Various losses are listed in the M-file

```matlab
% Fire Control Radar

r=(10:10:75000);  % Range (m)
sigmat = 1;  % Aircraft (sqm)
sigmas = 0.1;  % Sea skimmer (sqm)
dant = 1.5;  % Antenna Diameter (m)
pt = 75e03;  % Transmit power (W)
lamx = 30e-03;  % Wavelength (m)
lamk = 8.6e-03;

nfxdb = 3;  % Noise fig (dB)
nfkdb = 4.5;

tau = 133e-09;  % Pulse width (s)
k = 1.38e-23;  % Boltzmann (Js)
t = 290;
nintxdb = 24;  % Integ i mprove(dB)
nintkdb = 17;

ltxx = 3;  % Tx plumbing loss (dB)
ltxk = 4;

lscanx = 3.2;  % 2D Scan loss (dB)
lscank = 3.2;

lmiscx = 3;  % Misc loss (dB)
lmisck = 3;

alphax = 1;  % Atmos attn (dB/km) 0.02, 0.25, 1
alphak = 7;  % cREAR, 12.5, 25mm/hr 0.15, 3, 7
SNx = 13.6+10.4;  % S/N for Pd=0.95, Pfa=1.0e-06
SNk = 13.6+10.4;

% Calculate the antenna gain

gxdb = 10*log10(4*pi*0.7*pi*dant*dant/(4*lamx*lamx));
gkdb = 10*log10(4*pi*0.7*pi*dant*dant/(4*lamk*lamk));

% Propagation factor

propxdb = 10*log10(lamx*lamx/(4*pi).^3);
propkdb = 10*log10(lamk*lamk/(4*pi).^3);

% Transmit power

ptdb = 10*log10(pt);

% Target RCS

sigmatdb = 10*log10(sigmat);  % Aircraft

sigmasdb = 10*log10(sigmas);  % Sea skimmer

prxdb = ptdb+2*gxdb+propxdb+sigmasdb-ltxx-lscanx-lmiscx-40*log10(r)-
2*alphax*r/1000;
prkdb = ptdb+2*gkdb+propkdb+sigmasdb-ltxk-lscank-lmisck-40*log10(r)-
2*alphak*r/1000;

% Threshold for detection

thermxdb = 10*log10(k*t*0.672/tau);

thermkdb = 10*log10(k*t*0.672/tau);

threshxdb = (SNx + thermxdb + nfxdb --nintxdb)*ones(size(r));

threshkdb = (SNk + thermkdb + nfkdb --nintkdb)*ones(size(r));
semilogx(r,prxdb-threshxdb,r,prkdb-threshkdb);
grid

title('FIRE CONTROL RADAR: MAGNETRON: SKIMMER: Clear Air')
xlabel('RANGE (m)')
ylabel('SIGNAL LEVEL ABOVE Smin (dB)')
```
axis([0,100000,-50,200]);

lx=find((prxdb-threshxdb)<0);
lk=find((prkdb-threshkdb)<0);
min(r(lx))
min(r(lk))

Figure 15.33: Radar performance, clear air

Figure 15.34: Radar performance, 12.5mm/h rainfall
Table 15.15: System performance

<table>
<thead>
<tr>
<th>Target</th>
<th>Condition</th>
<th>X-Band Range (km)</th>
<th>Ka-Band Range (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft</td>
<td>Clear</td>
<td>58</td>
<td>37.5</td>
</tr>
<tr>
<td>RCS=1sqm</td>
<td>12.5mm/h</td>
<td>28.9</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>25mm/h</td>
<td>13.7</td>
<td>3.6</td>
</tr>
<tr>
<td>Sea Skimmer</td>
<td>Clear</td>
<td>34.4</td>
<td>25.8</td>
</tr>
<tr>
<td>RCS=0.1sqm</td>
<td>12.5mm/h</td>
<td>20.6</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>25mm/h</td>
<td>10.8</td>
<td>3.1</td>
</tr>
</tbody>
</table>

For aircraft detection, only the X-Band radar meets the specified range criteria of 15, 10 and 5km.

Both the X-Band and the Ka-Band radars meet the sea skimmer detection requirements of 6, 4 and 2km.

15.7.18. Effects of Multipath on Aircraft Detection

The following graphs show that the constructive and destructive interference caused by multipath can reduce the signal level $S_{\text{min}}$ to below 0dB under some circumstances.

The fades are more pronounced at X-Band where detection may not occur between 13 and 14km, and again at just below 10km.

None of the fades at Ka-Band dip so low.
Figure 15.36: Multipath effects on aircraft tracking at Ka-band

Figure 15.37: Multipath effects on sea skimmer tracking at X-band
15.7.19. Detection Threshold and CFAR

The false alarm rate is very sensitive to the setting of the detection threshold voltage. Because no velocity information is available, the radar must transmit a block of pulses and look in all the Doppler gates in all of the range gates over the designated range of the target.

Changes in radar characteristics with time (ageing) and changes in the target background characteristics mean that a fixed detection threshold is not practical. A Constant False Alarm Rate Processor (CFAR) is required.
We propose a cell-averaging CFAR processor that averages the returns from a particular Doppler bin across all of the range gates that span the designated range as shown in the highlighted row.

This averaging process excludes the cell under test which proceeds sequentially from gate 1 to gate N (it is shown in gate 3 in the Figure)

<table>
<thead>
<tr>
<th>Doppler Bins</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 15.40: Range/Doppler gate matrix showing cell containing target

**15.7.20. Transition to Track**

The mechanically operated FCR antenna moving at 60°/s (X-Band) or 200°/s (Ka-Band) during the search phase is unable to stop while the antenna is still pointing at the target.

The angles, range and Doppler bin at which detection takes place is recorded as the antenna sweeps past and the antenna returns to that designation more slowly.

If the target is not detected, then a slow search is conducted in angles while the range gates broaden their search until a detection occurs or the system times out. The angular rates of this new search are such that the antenna can stop while still illuminating the target, and a transition to track mode is made.

The process of detection involves closing the Doppler loop, followed by the range loop and finally the angle loop.

**15.7.21. Target Tracking**

A split tracking gate straddles the sum-channel range-gate in which the detection took place and early and late gate measurements are made only in the Doppler bin in which detection took place.

As the Doppler is generally ambiguous, it cannot be used to prime the range filter, so a long (typically 50ms or so) period is used to obtain a good range rate estimate to prime the range tracking filter.
The range tracking loop closes using early and late gate signals from the appropriate Doppler bin, and automatic tracking in range occurs using the difference between the early and late gate signals to keep the gates centred on the target.

Figure 15.41: Schematic diagram of range tracking configuration

The angle tracking loops are closed.

The azimuth and elevation error signals sampled simultaneously with the sum channel signal drive the angle servos to keep the antenna pointed at the target being tracked.

Target dynamics that dictate the real and apparent acceleration and tracking loop bandwidth determines the tracking accuracy.

Figure 15.42: Crossing target geometry
When viewed in radar polar co-ordinates, the target velocities will be

\[
\dot{R}_{\text{max}} = v_i, \quad \dot{A}_{\text{max}} = \frac{v_i}{R}, \quad \dot{E}_{\text{max}} = \frac{v_i}{R}.
\]

The real accelerations are

\[
\ddot{R}_{\text{max}} = a_i, \quad \ddot{A}_{\text{max}} = \frac{a_i}{R}, \quad \ddot{E}_{\text{max}} = \frac{a_i}{R}.
\]

The geometric accelerations and other derivatives are shown below. They are normalised in the angular co-ordinates to \( \dot{A}_{\text{max}} = \sigma_m = v_i / R_c \). Where \( R_c \) is the ground range at crossover.

In range, the derivatives are normalised to \( R_a = R_{\text{min}} \), at the point of closest approach.

Assuming that the target is travelling at 280m/s and flies past the radar at a height of 60m and a ground range of \( R_c = 60 \text{m} \), then the peak velocities and accelerations experienced (in polar co-ordinates)

\[
\sigma_m = 4.66 \text{rad/s} \quad \sigma_a = 13.75 \text{rad/s}^2 \quad \text{(from the graph)}
\]

\[
\sigma_e = 1.6 \text{rad/s} \quad \sigma_e = 10.42 \text{rad/s}^2
\]

\[
\dot{R}_{\text{max}} = 280 \text{m/s} \quad \ddot{R}_{\text{max}} = 923 \text{ m/s}^2
\]

Assuming that thermal noise and dynamic lag are the primary sources of error, then an optimum bandwidth is calculated as follows:

\[
B_n = \left[ \frac{a_i^2 k_m^2 f_r B \tau (S/N)}{1.6 \theta_3^2 R^2} \right]^{1/5}
\]

where: \( a_i \) – acceleration (real or geometric) (m/s\(^2\)) \[923\]
\( k_m \) – Monopulse Gain Constant (typ 1.6) \[1.6\]
\( f_r \) – Pulse repetition frequency (Hz) \[10\text{kHz}\]
\( B \tau \) - IF bandwidth and pulse width \[0.672\]
\( S/N \) – pulse signal to noise ratio \[10\]
\( \theta_3 \) – Antenna beamwidth (rad) \[0.024 \text{ or } 0.007\]
\( R \) – Crossing Range (m) \[85\]

The optimum angle servo bandwidth is 116Hz for the X-Band tracker and will be significantly higher for the Ka-Band unit. This is very high as the specified crossing range is extremely short.
If the bandwidth is expressed in terms of the equivalent noise bandwidth $B_n$ then the acceleration coefficient

$$K_a = 2.5B_n^2 = 0.63/t_o^2$$

Where $t_o$ is the equivalent averaging time for the tracking loop. The lag will be.

$$
\varepsilon_a = \frac{\dot{\sigma}_a}{K_a} = \frac{2\sigma_a}{2.5B_n^2} = \frac{13.75}{2.5 \times 116^2} = 0.409 \times 10^{-3} \text{ rad}
$$

$$
\sigma_a = \frac{\theta_3}{k_n \sqrt{2(S/N) f_r / B_n}} = \frac{0.024}{1.6 \sqrt{2 \times 10 \times 10^4 / 116}} = 0.36 \times 10^{-3} \text{ rad}
$$

The RMS tracking error caused by the thermal noise will be $\sigma_\theta = 0.36 \text{ mrad}$

The RMS tracking error caused by dynamic lag will be $\varepsilon_a = 0.41 \text{ mrad}$.

The alternative is to use a wide angle-servo bandwidth (≈200Hz) that will cope with the geometric accelerations, and to perform the filtering in Cartesian space with a bandwidth of between 1 and 4Hz that will cater for real target accelerations that should not exceed 6g for the aircraft.

![Figure 15.43: Tracking in Cartesian space](image)

### 15.8. References
